Non-linear non-radial oscillations of the self-gravitating disk taking into account halo

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В данной работе нами изучена численными методами эволюция нерадиальнных колебаний нестационарного диска, который окружен пассивным эллипсоидальным гало с однородной плотностью. С этой целью нами разработана специальная компьютерная программа. Проведены численные расчеты для различных значений параметров системы, такие как начальное возмущение, круговая скорость вращения диска и отношение массы гало к массе диска. Для каждого из возможных случаев, определена статистическая амплитуда, характеризующая степень деформации системы. Найдены критические значения параметров системы, при которых гало стабилизирует нелинейно нерадиальные колебания дисковой подсистемы галактик на ранней стадии их эволюции.

Ключевые слова: дискообразные галактики, гало, эволюция, нелинейные и нерадиальные колебания.

In this work, we have studied by numerical methods the evolution of non-radial oscillations of an unsteady disk, which is surrounded by a passive ellipsoidal halo with a uniform density. For this purpose, we have developed a special computer program. Numerical calculations are performed for various values of the system parameters, such as the initial perturbation, the circular speed of rotation of the disk, and the ratio of the halo mass to the disk mass. For each of the possible cases, a statistical amplitude is determined that characterizes the degree of deformation of the system. The critical values of the system parameters are found at which the halo stabilizes the non-linearly non-radial oscillations of the disk subsystem of galaxies at an early stage of their evolution.

Keywords: disk-shaped galaxies, halo, evolution, non-linear and non-radial oscillations.

I. Introduction

At present, even the most powerful telescopes do not provide an opportunity to directly observe the processes occurring at the earliest stages of the evolution of galaxies. Therefore, astronomers have not yet reliable information about the physics of these phenomena. In this regard, one of the urgent task of extra-galactic astrophysics and cosmology is now the study of the physics of the early non-stationary stage of galaxies and the problem of the formation of their subsystems (see, for example, [1-4] and references therein). Since almost half of the observed galaxies are disk-shaped, the study of the phenomena that took place in them at the very initial, non-stationary stage of their formation is a very interesting direction. It is especially interesting to find out how a galaxy's halo affects the evolution of its disk subsystem. Because the disk subsystem was formed after the halo and therefore the processes of its formation and evolution proceeded against the background of the halo (see eg [5-9] and references therein).

Proceeding from this, in this work we have investigated the problem of the evolution of the disk subsystem of galaxies taking into account the halo.

Obviously, we are not interested in questions about the constituent objects of the halo, which is reflected, for example, in [10-20]. Quite a lot of works have been published in the literature on the construction of equilibrium disk models taking into account the halo, as well as on the study of the stability of linear oscillations of the simplest models to reveal the role of the halo (see, for example, [21-24]. However, an analysis of a number of modern astrophysical observations shows that large-scale structures galaxies begin to form precisely at the non-linearly non-stationary stage of their evolution, when the processes of gravitational collapse of the medium, as well as its radial and non-radial global oscillations, etc. are still continuing. With this in mind, in [25] we considered the problem of constructing a non-linearly and non-radially oscillating disk model, which is surrounded by a passive ellipsoidal halo with a uniform density. Note that in the epoch when the disk has already formed in some form, the halo is obviously passive (see, for example, [1] and references therein). Also notice that the statement of the problem and preliminary results of the analysis of non-linear non-radial oscillations of a two-dimensional model without a halo in the general case are given in [26].

II. Basic formulas of the initial state

The problem we are considering presents has a certain difficulty, which is associated with the question of self-consistency. From the methodological point of view, there are two ways to take into account the halo. In the first of them, one should start with the compilation of the initial state taking into account the halo [27], only then impose a non-linear perturbation on the disk in the form of non-radial oscillations.

In the second method, the disturbance and the halo are simultaneously turned on. When "freezing" the imposed perturbation, we should obtain a different initial state in comparison with the noted equilibrium state. These two approaches ultimately give the same results, and therefore, for the sake of variety, in [25] we used the second method.

We have obtained a system of equations for the evolution of a non-radial oscillation of an unsteady disk in the presence of a halo in the form of matrix differential equations [25]:

$$\begin{cases} \dfrac{dD_1}{dt} = D_2, & \dfrac{dD_2}{dt} = -2D_1U - qAD_1, \\ \dfrac{dD_3}{dt} = D_4, & \dfrac{dD_4}{dt} = -2D_3U - qAD_3 \end{cases}, \tag{1}$$

where

$$U = N_2\left[E + (\Omega J - N_3)K^{-1}(\Omega J + N_3)\right]N_2^*, \tag{2}$$

and

$$K = E + N_3^2 + \Omega(JN_3 - N_3J)$$
$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{3}$$

The following notation was introduced in (1):

$$D_1 = N_2^{-1}, \quad D_2 = N_2^{-1}N_1,$$
$$D_3 = N_3N_2^{-1}, \quad D_4 = \frac{d}{dt}\left(N_3N_2^{-1}\right). \tag{4}$$

Moreover N_i, K and U are two-dimensional matrices [25], of which the matrices $N_1(t)$ and $N_3(t)$ are symmetric, the sign * means transposition. The parameter q is the ratio of the halo mass to the disk mass, and A is a known function [28] of the eccentricity of the halo meridional section. The following normalization is accepted everywhere [24]

$$\frac{4\pi}{3}G\rho_0 + \frac{\pi^2}{2R_0}G\sigma_0 = 1$$

where G is the gravitational constant, ρ_0 is the halo density, σ_0 and R_0 are the density and radius of the disk, respectively.

In order to integrate the system of matrix differential equations (1), we set physically acceptable initial conditions and for this we introduce the auxiliary matrix

$$Q = \begin{pmatrix} \mu & 0 \\ 0 & \mu^{-1} \end{pmatrix}, \tag{5}$$

such that $\vec{r} = Q\vec{r}_0$ and $\vec{v} = Q^{-1}\vec{v}_0$, where μ is the ratio of the perturbed semimajor axis to the unperturbed one ($\mu = a/a_0$), i.e. characterizes non-linear deviations from the unperturbed equilibrium state, for which $\mu = 1$. Then for the initial moment ($t = 0$) we have

$$(N_1)_0 \equiv 0, \quad (N_2)_0 \equiv Q^{-1}, \quad (N_3)_0 \equiv 0. \tag{6}$$

In order to determine the characteristics of the disk subsystem, the concept of statistical, average amplitude (the standard deviation of the area of the elliptical disk from the initial) was introduced:

$$\eta = \frac{1}{T}\int_0^T\left(\frac{S}{S_0} - 1\right)^2 dt = \frac{1}{T}\int_0^T (ab - 1)^2\, dt, \tag{7}$$

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where T is the integration interval, S is the area of the disk ($S = \pi ab$), moreover $a(t)$ and $b(t)$ are the semiaxes of the disk, S_0 is the area of the original unperturbed disk, the radius of which can be taken equal to unity.

III. Method of numerical analysis of the evolution equation for the disk of galaxies with allowance for the halo.

We have developed a method for the numerical analysis of the equation of non-linear non-radial oscillation of a self-gravitating disk in the presence of a halo (1). Here we are dealing with the Cauchy problem. To integrate the system of matrix differential equations (1), we must specify initial conditions. Based on this (5), taking into account (4) and (6), we obtain the following initial values:

$$\left(D_1\right)_0 = Q;\ \ (D_2)_0 = 0;\ \left(D_3\right)_0 = 0;\ \ (D_4)_0 = Q^{-1}. \quad (8)$$

For the major and minor semiaxes of the disk, we have the following values: $(a)_0 = \mu$ and $(b)_0 = \mu^{-1}$.

Now we determine the physically acceptable initial value for the function of the eccentricity of the meridional section of the halo.

When $e = 0$, then an uncertainty is obtained

$$A(e) = \frac{\sqrt{1-e^2}}{e^3}\left(\arcsin e - e\sqrt{1-e^2}\right)$$

When $e = 0$, there is an uncertainty of form 0/0. It is easy to expose this uncertainty by assuming

$$u(e) = \sqrt{1-e^2}\left(\arcsin e - e\sqrt{1-e^2}\right) =$$
$$= \sqrt{1-e^2}\arcsin e - e + e^3,$$
$$\vartheta(e) = e^3,$$

and calculate their derivatives:

$$u'''(e) = 6 - \frac{2}{1-e^2} - \frac{3e\arcsin e}{\sqrt{(1-e^2)^3}} - \frac{3e^2}{(1-e^2)^2} - \frac{3e^3\arcsin e}{\sqrt{(1-e^2)^5}}, \quad (9)$$
$$\vartheta'''(e) = 6.$$

Then using (9) we find:

$$A(0) = \frac{u'''(0)}{\vartheta'''(0)} = \frac{2}{3}. \quad (10)$$

Note that in numerical calculations it is more convenient to use the following notation for the statistical amplitude, η:

$$\eta = \frac{1}{N\delta t}\sum_{i=1}^{N}(a_i b_i - 1)^2\,\delta t = \frac{1}{N}\sum_{i=1}^{N}(a_i b_i - 1)^2. \quad (11)$$

This amplitude η can take values from zero (in the absence of deformations, i.e. when the area of the perturbed disk during evolution remains equal to the area of the unperturbed disk) up to infinitely large values (at large deformations that destroy the disk, i.e. when the semiaxes of the disk a and $b \to \infty$ at $t \to \infty$).

With the help of our computer program, numerical calculations were carried out for various values of the parameter q, the initial perturbation $\mu \in [1;2]$ and the circular speed of rotation of the disk $\Omega \in [0;1]$. It should be noted here that numerical calculations can be controlled with using simple, known invariants of the non-linear evolution of the system. For various values of q, μ and Ω we have determined the statistical amplitude η, characterizing the degree of deformation of the system. The dependence of the statistical amplitude on the initial perturbation $\eta(\mu)$ and the rotational speed $\eta(\Omega)$ is obtained. It should be noted that the values of the statistical amplitude have a fairly wide range from 10^{-7} to 10^5, so it is more expedient to switch to a logarithmic scale.

IV. Results of numerical calculations

When the numerical calculations were carried out for different values of the disk rotation parameter Ω and the initial perturbation μ for the case q=0, it was found that at moderate values of the perturbations μ<1.15, the behavior of the logarithm versus amplitude $\lg\eta(\mu)$ for stable $\Omega \in [0;0.5]$ and unstable $\Omega \in [0.6;0.8]$ regions with respect to the parameter of disk rotation differ greatly. It is also received that at large values of perturbations μ the curves behave identically, thus starting from μ=1.15, the oscillations of the disk model become unstable and at $\mu \geq 1.8$ the disk disintegrates and dissipates, because the disk semiaxes $a(t)$ and $b(t)$ grow linearly with time. Note that with an increase in the integration interval, the overall picture changes insignificantly.

To determine the boundary of stable and unstable oscillations from the parameter of disk rotation, separate numerical calculations were performed. The results obtained show that at large values of disturbances $\mu \geq 1.20$ no dependence is felt, which also confirms the instability of such oscillations. At small

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values of perturbations, deformations change insignificantly in the range $\Omega \in [0;0.3]$, and in the range $\Omega \in [0.4;0.5]$ we have a zone of reduced deformations, but at $\Omega=0.5$ a sharp increase is observed, and the value of the statistical amplitude η increases by several orders of magnitude.

Thus, the results obtained showed a qualitative change in the nature of oscillations in stable and unstable zones of values for the parameters μ and Ω. In the case of stable oscillations, $\Omega<0.5$, the disk semiaxes $a(t)$ and $b(t)$ are strictly periodic functions of time with a constant or varying amplitude pool fluctuations at small perturbations μ. At $\Omega=0$ with an increase μ up to 1.5, and at $\Omega=0.4$ μ increase up to 1.3, the behavior of the disk semiaxes does not differ from the behavior of $a(t)$ and $b(t)$ of the instability zone $\Omega>0.5$. In these cases, although it is possible to trace certain pulsations of the disk, they have not a strict periodicity, and the amplitude of the pulsations is several times greater than the value of the initial disturbance μ.

The calculations of the evolution of the disk model were also carried out in the time interval $[0; 50\pi]$ with a relative accuracy of integration $\Delta = 10^{-7}$. The values $a(t)$ and $b(t)$ were reminded at intermediate points $t_i = \pi i/20$. We calculated the evolution of the disk for various combinations of the values of the main parameters q, Ω and μ. An analysis of the results of numerical calculations shows that for $q=1$ and $0\le\Omega\le1$, the halo stabilizes non-linear non-radial oscillations of the disk, but starting from values $q\le0.001$ it ceases to stabilize these oscillations.

V. Conclusion

Let us list the main results obtained by us in this work.

A method has been developed for the numerical analysis of the system of equations for the evolution of non-linearly non-radially oscillations of a self-gravitating disk taking into account the halo.

The time dependences of the major and minor semiaxes of the disk were obtained for various values of the parameters of the system.

The critical values q and the disk rotation parameter Ω are determined, at which the halo stabilizes non-linear non-radial oscillations of the disk subsystem of galaxies.

The statistical amplitude η characterizing the degree of deformation of the system was found and it was found that in the range of the rotation parameter $\Omega \in [0;0.3]$ at small perturbations of deformation its values change insignificantly, and in the range $\Omega \in [0.4;0.5]$ there is a zone of reduced deformations, but at $\Omega=0.5$, a sharp increase of η is observed, its value increases by several orders of magnitude.

It is proved that for large μ the curves of the disk semiaxes behave identically and, starting with $\mu=1.15$, the oscillations of the disk model become unstable and for $\mu\ge1.8$ the disk semiaxes grow linearly with time and the disk disintegrates (dissipates).

It was found that at $q=1$ and $0\le\Omega\le1$, the halo stabilizes non-linear non-radial oscillations of the disk, but starting from values $q\le0.001$ it ceases to stabilize these oscillations.

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Галони ҳисобга олган холда ўзгравитацияланувчи дискнинг ночизиқли норадиал тебраниши

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Мазкур ишда биз бир жинсли пассив эллипсоидал гало билан ўралган ностационар дискнинг норадиал тебраниши эволюциясини рақамли усул ёрдамида ўргандик. Шу мақсадда, ҳисоб-китоблар учун махсус компьютер программаси ишлаб чиқилди. Тизим параметрларининг, яъни бошланғич ғалаёнланиш, дискнинг бир бутун айланиш тезлиги ва гало массасининг диск массасига насбатининг турли қийматлари учун рақамли ҳисоб-китоблар олиб борилди. Ҳар бир ҳол учун, хусусан, система деформацияси даражасини характерловчи статистик амплитуда аниқланди. Система параметрларининг, галактикалар эволюциясининг бошланғич босқичида гало диск подструктуранинг ночизиқли норадиал тебранишини стабилловчи, критик қийматлари аниқланди.

Калит сўзлар: дисксимон галактикалар, гало, эволюция, ночизиқли норадиал тебранишлар.

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