On time evolution of force-free magnetospheres around a slowly rotating compact object

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В этой работе переформулирована структура устойчивости бессиловой магнитосферы вращающейся чёрной дыры, которая обсуждалась в работе [[1]. Мы вносим поправки в основные уравнения магнитогидродинамики, написанные автором указанной работы. Показано, что в режиме медленного вращения уравнения (18)-(20) и уравнения Максвелла (B6), (B7), и (B13) в работе [1] содержали неточности. Несмотря на большое количество опечаток в уравнениях в работе [1], наши вычисления показывают, что это не нарушает структуру устойчивости бессиловой магнитосферы, что, в свою очередь, означает, что механизм извлечения энергии посредством устойчивой бессиловой магнитосферы черной дыры может существовать.

Ключевые слова: чёрная дыра, бессиловая магнитосфера, извлечение энергии.

This paper reformulates the stability structure of force-free magnetosphere of a rotating black hole discussed in paper [1]. We do some corrections in the main equations of relativistic magnetohydrodynamics written by this author. It is shown that in a regime of slow rotation, evolution equations (18)-(20) and Maxwell equations (B6), (B7), and (B13) of paper [1] were not written correctly. Although there is a large number of typos in the equations given by [1], our calculations show this does not violate the stability structure of force-free magnetosphere which in its turn means that the energy extraction mechanism via surrounding, stable force-free magnetosphere of a rotating black hole immersed in magnetic field can exist.

Keywords: black hole, force-free magnetosphere, energy extraction.

I. Introduction

It is widely believed that a rotating black hole most likely powers compact astrophysical systems such as active galactic nuclei, gamma-ray bursts, and black hole x-ray binaries. Theoretical arguments support an idea that when a supermassive black hole with a mass greater than which is thought to exist at the center of most of galaxies in the Universe accretes material, a substantial fraction of the gravitational binding energy of the infalling matter can be radiated away (see [2], for review). The mechanism for removing black hole spin energy was proposed by [3] which is one of the more plausible theories of energy extraction. In that paper they imagined that as a black hole accretes, it would inevitably trap some net magnetic flux due to accreted field lines with connections to infinity. There would then be an approximately time-steady magnetic field configuration with field lines embedded in the black hole's event horizon, even while their far ends close at very large distance from the black hole, the enforced rotation of space-time due to the black hole spin would then drive an MHD wind. A number of extensive simulations of the energy exraction also exist, such as [4, 5]

There are several issues regarding this model that whether this mechanism physically is allowed to

work. Some of the unresolved or physically not fully understood problems in recent days are the nature of the load in the Blandford-Znajeck model, the causality and stability of a force-free magnetosphere, global current closure, and the role of boundary conditions on the channels through which the extracted energy is released. One of the first issues was raised by [6]: whether the black hole event horizon and plasma far from the black hole could be in causal contact. They argued that, though the plasma density is taken to be negligible in the usual formulation of the Blandford-Znajeck mechanism, it can not be literally equal to zero. They have shown that there would inevitably be a surface surrounding the event horizon within which the inward velocity of the plasma would be greater than the fast magnetosonic speed, and no signal carrying energy or information could propagate outward across that surface. Later [7] analyzing force-free waves and black hole magnetosphere causality concluded that charge and current perturbations cannot be transported along the fast-mode characteristics in the force-free limit.

A major components of the active galactic nuclei engine is thought to be a magnetosphere surrounding supermassive black holes [8, 9, 10, 11]. The question that whether such black holes can communicate stresses to a distant load via a surrounding, force-free magnetosphere, and whether such structures are stable has been studied recently by [1]. It was shown by [2] that the conclusive results of [1] fail when one deeply discuss them. According to [1] fast wave can propagate at the speed of light with which means lightlike plane wave solutions can attain a longitudinal polarization. This was in controversy with simulations of Komissarov on polarization information [13, 14]. By studying the global physical properties of a force-free degenerate electrodynamics he tried to resolve this controversy and it was shown that force-free fast waves are not able to propagate changes in $E_{\parallel}$.

It is worth noting that alternative models of energy extraction from black holes also exist, such as the so called magnetic Penrose process [15, 16, 17]. It is the generalization of a Penrose process taking into account the magnetic field of the central object. So-called moderate regime of MPP is realized when the charged infalling particle decays into charged fragments and this regime of MPP has a direct analogy to Blandford-Znajek mechanism [16]. There is also an ultra-efficient regime of the MPP which arises when the infalling particle is neutral and it splits into charged fragments. This regime might serve as a plausible explanation of the extragalactic high-energy component of cosmic radiation [15].

Studying the second part of the paper of [1] we have found many mathematical typos in the set of Maxwell equations, and force free condition in open form (Appendix B of [1]). The investigation on spacetime-driven evolution of a force-free magnetosphere thus could bring to inaccurate results. Following this paper we reformulate all formulae given by this author and try to obtain more exact solution of evolution equations.

The paper is organized as follows. First of all, in Section 2 the azimuthal magnetic potential is determined using vacuum solution of general relativistic Maxwell equations. Then in the force-free constraint it is found the evolution equation for the electromagnetic field. Using expansion method the solution of general relativistic evolution equations are and at least in Section 3 some concluding remarks are given.

Throughout, we use a space-like signature as (–, +, +, +) and a system of units.

II. Spacetime driven evolution of a force-free magnetosphere

Consider a compact gravitating object, initially nonrotating and having static axially symmetric magnetic field. The exterior gravitational field of the compact object is described by the background metric (30). Suppose that initially magnetic field is defined by the vacuum solution of Maxwell equations (32)–(39) which is given by the vector potential $A_\alpha\left\{0,0,0,A_\phi\right\}$.

By definition the magnetic field measured by an observer with four velocity u_α is $B^\alpha = -1/2\eta^{\alpha\beta\mu\nu}u_\beta F_{\mu\nu}$, where $\eta^{\alpha\beta\mu\nu}$ is pseudotensorial expression for Levi-Civita symbol, $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ is the tensor of electromagnetic field. In the case of axial symmetry, the poloidal magnetic field measured by the proper observer in the metric (30) can be expressed through azimuthal component of the vector potential as

$$B^{\hat{r}}_p\left(t,r,\theta\right) = \frac{1}{r^2\sin\theta}\partial_\theta A_\phi = \sum_{\ell=1}^{\infty}\frac{1}{r^2}b_\ell(t,r)P_\ell(\cos\theta), \tag{1}$$

where the useful relation (equation (A2) of [18]) for

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the angular derivatives of the Legendre function P_ℓ has been used.

$$B_p^{\hat{\theta}}(t,r,\theta) = -\frac{N}{r\sin\theta}\partial_r A_\phi = \\ = \sum_{\ell=1}^{\infty}\frac{1}{r}\frac{N\sin\theta}{\ell(\ell+1)}\partial_r\left(b_\ell(t,r)\right)\partial_\theta P_\ell(\cos\theta), \quad (2)$$

$$\frac{1}{\sin\theta}\partial_\theta\left[\sin\theta\partial_\theta\left(P_\ell(\cos\theta)\right)\right] = -\ell(\ell+1)P_\ell(\cos\theta). \quad (3)$$

Then the Maxwell equations for nonvanishing component of vector potential (39) will take form

$$r^2\partial_r\left(N^2\partial_r A_\phi\right) + \sin\theta\partial_\theta\left(\frac{1}{\sin\theta}\partial_\theta A_\phi\right) = 0. \quad (4)$$

By using the expansion of vector potential in Legendre function for the dipolar case (when $\ell = 1$) and introducing a new variable $A_\phi = r^2 F\sin^2\theta$ one can show that equation (4) takes form Eq. (87) of [18]).

$$\frac{d}{dr}\left[N^2\frac{d}{dr}(r^2F)\right] - 2F = 0, \quad (5)$$

which has a solution given by

$$A_\phi(r) = Fr^2\sin^2\theta = \frac{3\mu r^2}{8M^3}\left[\ln N^2 + \frac{2M}{r}(1+\frac{M}{r})\right]\sin^2\theta \quad (6)$$

and reduces to a flat spacetime solution for a dipole with magnetic moment μ, i.e.

$$\lim F(r) = \frac{\mu}{r^3}. \quad (7)$$

It is necessary to underline that in paper [1] the solution for $A_\phi = (B/2)r^2\sin^2\theta$ does not satisfy to the Maxwell equations((B6) – (B9) of [1]).

Imagine now that the state of the gravitating object is perturbed adiabatically, such that it slowly acquires angular momentum and the spacetime could be described by the metric (30). From equation (42) it is clear that the azimuthal current $\left(J^{\hat{\phi}} + \frac{\omega r\sin\theta}{N}J^{\hat{t}}\right)$ is of order $O(a^2)$ and that to the second order in a the force-free condition (40 and 41) reduce to

$$J^{\hat{r}} = -\frac{1}{Nr}\frac{\left(\partial_\theta A_\phi\right)}{\left(\partial_r A_\phi\right)}J^{\hat{\theta}} = -Y(r)\cot\theta J^{\hat{\theta}}, \quad (8)$$

$$E^{\hat{\theta}} = \frac{1}{Nr}\frac{\left(\partial_\theta A_\phi\right)}{\left(\partial_r A_\phi\right)}E^{\hat{r}} = Y(r)\cot\theta E^{\hat{r}}, \quad (9)$$

where

$$Y(r) = \frac{r}{NM}\left[\ln N^2 + \frac{2M}{r}\left(1+\frac{M}{r}\right)\right]\times \\ \times\left[\frac{r}{M}\ln N^2 + \frac{1}{N^2} + 1\right]^{-1}. \quad (10)$$

From equations (32), (35) and (9) one can get the evolution equation for the toroidal magnetic field

$$\frac{\partial B^{\hat{\phi}}}{\partial t} = -\frac{N}{r}\left(rNE^{\hat{\theta}}\right)_{,r} + \frac{N}{r}Y^{-1}(r)\left(\tan\theta E^{\hat{\theta}}\right)_{,\theta} - \\ -\frac{9aN\mu}{2M^2r^3}\left[\ln N^2 + \frac{2M}{r}\left(1+\frac{M}{r}\right)\right]\sin\theta\cos\theta. \quad (11)$$

From equations (37), (38) and (8), (9) we obtain a differential equation for the electric field $E^{\hat{\theta}}$.

$$\left\{\left(1+Y^{-2}(r)\tan^2\theta\right)\right\}\frac{\partial^2 E^{\hat{\theta}}}{\partial t^2} + \frac{N}{r}\left[r\ N\left(\partial_t B^{\hat{\phi}}\right)\right]_{,r} - \\ -\frac{N}{r\cos\theta}Y^{-1}(r)\left[\sin\theta\left(\partial_t B^{\hat{\phi}}\right)\right]_{,\theta} = 0. \quad (12)$$

Then in the force-free magnetosphere the equation of the continuity of the electric charge $J^{\alpha}_{;\alpha}$ is given by

$$\partial_t J^{\hat{t}} + \frac{N}{r^2}\partial_r\left(r^2NJ^{\hat{r}}\right) + \frac{N}{r\sin\theta}\partial_\theta\left(\sin\theta J^{\hat{\theta}}\right) = 0. \quad (13)$$

Linearizing equation (39), one can derive wave equation for the perturbation of the vector potential δA_ϕ

$$\partial_t^2\delta A_\phi - N^2\partial_r\left(N^2\partial_r\delta A_\phi\right) - \frac{N^2\sin\theta}{r^2}\partial_\theta\left(\frac{1}{\sin\theta}\partial_\theta\delta A_\phi\right) = \\ = 4\pi N^2 r\sin\theta\left(J^{\hat{\phi}} + \frac{\omega r\sin\theta}{N}J^{\hat{t}}\right), \quad (14)$$

where the right hand side of the equation is proportional to a^2 and is negligible in our approximation.

Define now the dimensionless time $\tau = t/2M$. Near the horizon, the dimensionless variable $x = N^2$ is

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small and equation (12) can be solved using perturbation approach. It is necessary to underline the force-free approximation could be violated in the region very close to the event horizon i.e. when is quite small and plasma inertia and gravity start becoming important for the particle's dynamics. However as it was mentioned in [19] that it is possible to effectively extend the domain of the validity of the force-free approximation almost up to the horizon.

Near the horizon the ratio

$$\lim_{r\to 2M}\left[\ln N^2+\frac{2M}{r}\left(1+\frac{M}{r}\right)\right]^{-1}\times \\ \times\left[\frac{r}{M}\ln N^2+\frac{1}{N^2}+1\right]=\left(\frac{1}{xb}+1\right) \tag{15}$$

may play a significant role, where $b = \ln(x)+3/2$. It is clear that at a some fixed point $r = 2M(1+x)$, function $F(r)=(3\mu b)/(8M^3)$ varies slowly with x, and one may choose it to be F=const<0. We suppose that the solution can be expanded as series

$$\tilde{E}^{\hat{\theta}}=\sum_n f_n(\tau)x^n P_l^m(\cos\theta)\approx \\ \approx\left(f_0(\tau)+f_1(\tau)x\right)3\cos\theta\sin\theta+\mathrm{O}\left(x^2\right) \tag{16}$$

with f_n as unknown functions, where for our purpose we choose $l = 2$, $m = 1$. Then the evolution equations (12) and (11) take more simple form

$$\frac{\partial\tilde{B}^{\hat{\phi}}}{\partial t}=-\frac{1}{2M}\times \\ \times\left(\tilde{E}^{\hat{\theta}}+x\tilde{E}^{\hat{\theta}}_{,x}-\frac{1}{b}\tilde{E}^{\hat{\theta}}-x\tilde{E}^{\hat{\theta}}+\frac{Fa}{M}3\sin\theta\cos\theta\right), \tag{17}$$

$$\frac{\partial^2\tilde{E}^{\hat{\theta}}}{\partial t^2}+\frac{1}{4M^2}\left\{\begin{array}{l}-\tilde{E}^{\hat{\theta}}-3x\tilde{E}^{\hat{\theta}}_{,x}+\\ +3x\tilde{E}^{\hat{\theta}}+\frac{1}{b}(2-3x)\tilde{E}^{\hat{\theta}}+\\ +\frac{2x}{b}\tilde{E}^{\hat{\theta}}_{,x}-\frac{1}{b^2}\tilde{E}^{\hat{\theta}}+\\ +\frac{Fa}{M}3\sin\theta\cos\theta\left(3x-1+\frac{1}{b}\right)\end{array}\right\}=0, \tag{18}$$

where $E^{\theta}=N^{-1}E^{\theta}$ and $B^{\phi}=N^{-1}B^{\phi}$ are new variables. In the above equations we neglected all small terms which depend on x^2 and have used the assumption (16).

Hereafter we assume that the specific angular momentum changes with time as $a(\tau)=a_0(1-e^{-\tau/T})$, where the dimensionless parameter $\tau=t/2M$ is introduced, T is the timescale in units of $2M$.

To zeroth order we have

$$E^{\theta}=3f_0(\tau)\cos\theta\sin\theta, \tag{19}$$

where function f_0 and the toroidal magnetic field are governed by the following equations

$$\frac{\partial^2 f_0}{\partial\tau^2}-\left(1-\frac{2}{b}+\frac{1}{b^2}\right)f_0-\left(1-\frac{1}{b}\right)\frac{Fa}{M}=0, \tag{20}$$

$$\frac{\partial B^{\hat{\phi}}}{\partial\tau}=-N\left(f_0\left(1-\frac{1}{b}\right)+\frac{Fa}{M}\right)3\sin\theta\cos\theta, \tag{21}$$

which is found from the equations (17) and (18). From these equations one can see that in contrast to the results of [1] (see their equation (23)), there is no additional $-2M$ term in the denominator on the the right hand side and the equation itself is rather complex. To zeroth order the solution of function f is

$$f_0(\tau)=\frac{-Fa_0b\left(T^2-1\right)b^2-2T^2b+T^2}{M(b-1)\left[(T+1)b-T\right]\left[(T-1)b-T\right]}- \\ -\frac{-T^2(b-1)^2e^{-\tau/T}+b^2e^{(1-b)\tau/b}}{M(b-1)\left[(T+1)b-T\right]\left[(T-1)b-T\right]}, \tag{22}$$

which satisfies the initial conditions $E^{\hat{\theta}}=\partial_\tau E^{\hat{\theta}}=0$ at time τ=0. Here we have to mention that near the horizon F takes negative values (see Fig. 1). To first order from the equation (18) we obtain

$$\frac{\partial^2 f_1}{\partial\tau^2}+\left(1-\frac{1}{b}\right)(3f_0-4f_1)-\frac{1}{b^2}f_1+\frac{3Fa}{M}=0, \tag{23}$$

which has the solution (see Fig. 1).

$$f_1(\tau)=3b^3Fa_0\left\{\begin{array}{l}bT^2(3b-2)e^{-\tau/T}-\\ \left(4b^2T^2-b^2-4T^2b+T^2\right)e^{-\tau+\tau/b}\\ -\left(b^2\left(1-T^2\right)+T^2(2b-1)\right)e^{(1-2b)\tau/b}\end{array}\right\}\times \\ \times\{M(3b-2)(T^2\left(b-1\right)^2-b^2)(T^2\left(b-1\right)^2-b^2)\}^{-1} \\ \ldots.(24)$$

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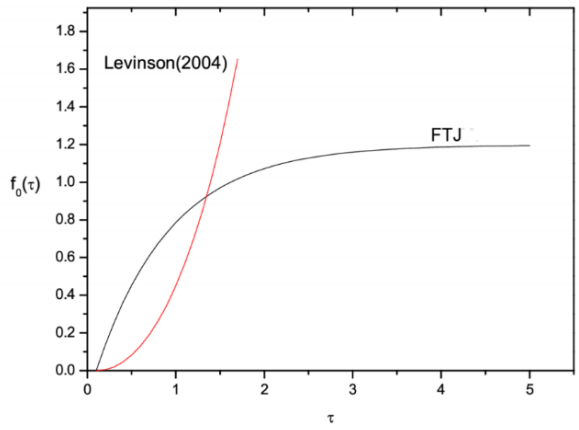


Figure 1. Zeroth order of the evolution of expansion function $f(\tau, x)$ with time, where for convenience the mass and momentum, are taken to be unit, for $x = 0.001$, $T = 0.01$, and angular momentum is taken to be $a_0 = 0.7$.

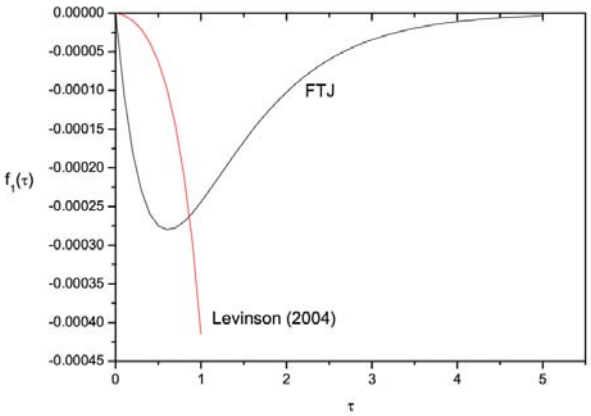


Figure 2. First order of the evolution of expansion function $f(\tau, x)$ with time, where for convenience the mass and momentum, are taken to be unit, for $x = 0.001$, $T = 0.01$, and angular momentum is taken to be $a_0 = 0.7$.

It is clear from the behavior of functions $f_0(\tau)$ and $f_1(\tau)$ that at the initial time $f_1(\tau)$ is very small compared to $f_0(\tau)$, and is evolving very slow with time.

At first, $f_1(\tau)$ grows up exponentially with negative value then, suddenly the evolution falls down to zero which means after that time the above solution is no longer valid. Note in contrast to results of [1], evolution of electric field is more complex, but despite of complexities the results gives us the steady state solution, i.e. the evolution is adiabatic and higher orders do not give significant contribution. However comparing figures we find that it is not clearly shown in [1] that the evolution really reaches the steady point.

Neglecting first order terms for toroidal magnetic field to zeroth order using equation (21) and expansion function (22) one can obtain

$$B_0^{\phi}(\tau) = \frac{Na_0b^2F\left[\frac{b}{b-1}\left(1-e^{-(b-1)\tau/b}\right)-T\left(1-e^{-\tau/T}\right)\right]}{M(bT-b-T)(bT+b-T)}\times \times 3\sin\theta\cos\theta, \tag{25}$$

which evolves adiabatically with time (see Fig. 2).

To zeroth order there is no contribution to the poloidal currents, but to the first order near the rotation axis, that is at small angles using equations (11), (12), (37) and relation (9) we find

$$j_1^{\hat{r}} = -\frac{3a_0b^2F\cos^2\theta\left(\frac{b}{b-1}e^{(1-b)\tau/b}-Te^{-\tau/T}\right)}{8\pi M^3(bT-b-T)(bT+b-T)}x. \tag{26}$$

It is clear that poloidal current initially generated everywhere in space as shown by [20], that is equal to

$$j^{\hat{r}}(\tau=0) = \frac{3a_0b^2F\cos^2\theta}{8\pi M^3(b-1)(bT+b-T)}x. \tag{27}$$

But for toroidal currents one can find from (36) that to zeroth order it is equal to

$$j_0^{\hat{t}} = \frac{3f_0\sin\theta\cos\theta}{4\pi M}, \tag{28}$$

and to first order

$$j_1^{\hat{t}} = \frac{3bf_0+(1+b/4)f_1}{4\pi M}\times 3x\sin\theta\cos\theta. \tag{29}$$

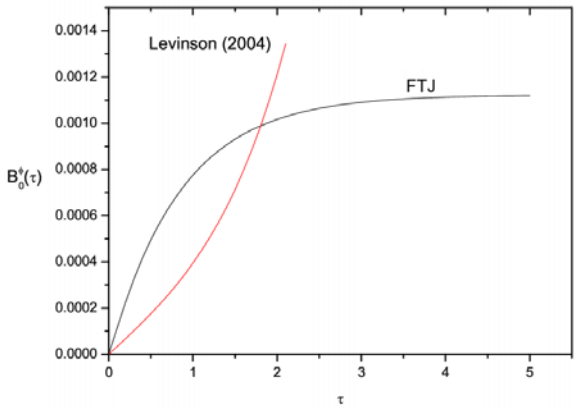


Figure 3. Evolution of toroidal magnetic field with time to zeroth order near the equatorial plane at θ=0.01 radian, where for convenience the mass and momentum, are taken to be unit, for x=0.001, T=0.01, and angular momentum is taken to be a_0=0.7.

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The above solutions are rather different from solutions given by [1], for example, there is an additional term M/π in equations (26), (27), which should be in the current expressions in order to have correct units. Though the evolution equations are also different, and rather complicated than that presented in the paper by Levinson, but this does not cause to break the evolution structure and one can ensure that in some early times the toroidal magnetic field evolves close to its steady state value. The above results confirm that the black hole magnetosphere is stable, despite the lot of typos observed in the paper [1].

III. Conclusion

In this paper it was shown that Maxwell equations (B6), (B7), the homogeneous Maxwell equation (B13) and the force-free condition (B14-B17) of [1] to the second order in $O(a^2)$ do not coincide with equations obtained by us. It may cause to obtain inaccurate results on the stability structure formulation. It is also not clear from the solutions in paper [1] that poloidal electric field and toroidal magnetic field are evolving adiabatically with time (solutions show us exponentially growth with time). Obtaining more accurate solutions we can conclude that in fact poloidal electric field and toroidal magnetic fields grow adiabatically. We reformulated all formulae obtained by this author and conclude that although the results are rather different, but this does not violate the stability structure, i.e. after sometime the toroidal magnetic field and the charge density $j^{\hat{t}}$ evolve to their steady state solution. The above suggestion confirm that the black hole magnetosphere is stable [21].

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Appendix. Electrodynamic equations of force-free magnetosphere

In a coordinate system (ct, r,θ, ϕ) the geometry of spacetime around "slowly rotating" gravitating object is described by

$$ds^2 = -N^2dt^2 + N^{-2}dr^2 - 2\omega(r)r^2\sin^2\theta dtd\phi + \\ +r^2d\theta^2 + r^2\sin^2\theta d\phi^2 , \quad (30)$$

where $N = 1–2M/r$ is the lapse function and $\omega(r)$ can be interpreted as the angular velocity of a free falling (inertial) frame and is also known as the Lense-Thirring angular velocity.

The radial dependence of ω is

$$\omega(r) \equiv \frac{d\phi}{dt} = -\frac{g_{0\phi}}{g_{\phi\phi}} = \frac{2J}{r^3}, \quad (31)$$

where $J = aM$ is the total (a is the specific) angular momentum of metric source with mass M as measured from infinity.

Maxwell equations are

$$N\sin\theta\left(r^2B^{\hat{r}}\right)_{,r} + r\left(\sin\theta B^{\hat{\theta}}\right)_{,\theta} + rB^{\hat{\phi}}_{,\phi} = 0, \quad (32)$$

$$\left(r\sin\theta\right)\frac{\partial B^{\hat{r}}}{\partial t} = N\left[E^{\hat{\theta}}_{,\phi} - \left(\sin\theta E^{\hat{\phi}}\right)_{,\theta}\right] - \left(\omega r\sin\theta\right)B^{\hat{r}}_{,\phi}, \quad (33)$$

$$\left(\frac{r\sin\theta}{N}\right)\frac{\partial B^{\hat{\theta}}}{\partial t} = -E^{\hat{r}}_{,\phi} + \sin\theta\left(rNE^{\hat{\phi}}\right)_{,r} - \left(\frac{\omega r\sin\theta}{N}\right)B^{\hat{\theta}}_{,\phi}, \quad (34)$$

$$\left(\frac{r}{N}\right)\frac{\partial B^{\hat{\phi}}}{\partial t} = -\left(rNE^{\hat{\theta}}\right)_{,r} + E^{\hat{r}}_{,\theta} + \sin\theta\left(\omega r^2B^{\hat{r}}\right)_{,r} + \frac{\omega r}{N}\left(\sin\theta B^{\hat{\theta}}\right)_{,\theta} \quad (35)$$

and

$$N\sin\theta(r^2E^{\hat{r}})_{,r} + r(\sin\theta E^{\hat{\theta}})_{,\theta} + rE^{\hat{\phi}}_{,\phi} = 4\pi r^2\sin\theta J^{\hat{t}}, \quad (36)$$

$$N\left[\left(\sin\theta B^{\hat{\phi}}\right)_{,\theta} - B^{\hat{\theta}}_{,\phi}\right] - \left(\omega r\sin\theta\right)E^{\hat{r}}_{,\phi} = \\ = \left(r\sin\theta\right)\frac{\partial E^{\hat{r}}}{\partial t} + 4\pi Nr\sin\theta J^{\hat{r}}, \quad (37)$$

$$B^{\hat{r}}_{,\phi} - \sin\theta\left(rNB^{\hat{\phi}}\right)_{,r} - \left(\frac{\omega r\sin\theta}{N}\right)E^{\hat{\theta}}_{,\phi} = \\ = \left(\frac{r\sin\theta}{N}\right)\frac{\partial E^{\hat{\theta}}}{\partial t} + 4\pi r\sin\theta J^{\hat{\theta}}, \quad (38)$$

$$\left(NrB^{\hat{\theta}}\right)_{,r} - B^{\hat{r}}_{,\theta} + \sin\theta\left(\omega r^2E^{\hat{r}}\right)_{,r} + \frac{\omega r}{N}\left(\sin\theta E^{\hat{\theta}}\right)_{,\theta} = \\ = \left(\frac{r}{N}\right)\frac{\partial E^{\hat{\phi}}}{\partial t} + 4\pi rJ^{\hat{\phi}} + \frac{4\pi\omega r^2\sin\theta}{N}J^{\hat{t}}. \quad (39)$$

The force-free condition $F_{\mu\nu}J^{\nu} = 0$ in the axial symmetric magnetosphere will take form

$$NE^{\hat{r}}J^{\hat{r}} + NE^{\hat{\theta}}J^{\hat{\theta}} - \frac{1}{r\sin\theta}\left(\partial_t A_{\phi}\right)\left(J^{\hat{\phi}} + \frac{\omega r\sin\theta}{N}J^{\hat{t}}\right) = 0, \quad (40)$$

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$$N^{-1}\left(\partial_t A_\phi\right)J^{\hat{t}} + \left(\partial_r A_\phi\right)NJ^{\hat{r}} + \frac{1}{r}\left(\partial_\theta A_\phi\right)J^{\hat{\theta}} = 0, \quad (41)$$

$$N^{-1}E^{\hat{r}}J^{\hat{t}} + \frac{1}{r\sin\theta}\left(\partial_r A_\phi\right)\left(J^{\hat{\phi}} + \frac{\omega r\sin\theta}{N}J^{\hat{t}}\right) + \\ + \frac{1}{N}B^{\hat{\phi}}J^{\hat{\theta}} = 0 \quad . \quad (42)$$

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Секин айланувчи компакт объектнинг атрофида кучдан ҳоли магнитосферанинг вақт эволюцияси

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Ушбу ишда [1] мақолада муҳокама қилинган айланувчи қора ўра кучдан холи магнитосферасининг турғунлик структураси қайта формаллаштирилди. Биз юқоридаги иш муаллифи ёзган магнитогидродинаминканинг асосий тенгламаларини тўғирладик. Секин айланиш режимида [1] ишдаги (18)-(20) тенгламалар ҳамда (B6), (B7) ва (B13) Максвелл тенгламаларида хатоликлар борлиги кўрсатилди. Тенгламаларда кўп сонли хатоликлар мавжудлигига қарамай, бизнинг ҳисоб-китобларимиз кўрсатишича, кучдан холи магнитосферанинг турғунлик структураси бузилмайди, бу эса, ўз навбатида, қора ўрадан уни ўраб турувчи турғун кучдан холи магнитосфера орқали энергия олиш механизми мавжуд бўла олишини англатади.

Калит сўзлар: қора ўра, кучдан ҳоли магнитосфера, энергия олиш.

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