Possible fractional quantum Hall effect and Wigner crystal states in two-dimensional Coulomb gases as a result of anyon correlations

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В нашей недавно опубликованной работе мы вывели приближенную аналитическую формулу для энергии основного состояния (2D) газа из заряженных энионов. Этот результат позволил нам в настоящей работе оценить обменно-корреляционную энергию, которая, как хорошо известно из литературы, является разностью энергии основного состояния и энергии Хартри-Фока для фермионов с энионным параметром ν=1, и сформулировать энионную-корреляционную энергию для остальных ν<1. Минимумы последней для всех фиксированных значений ν, точно также как и минимумы для энергии основного состояния при тех же ν, являются функциями только ν, что означает, что они являются зависимыми только от энионной статистики без влияния эффекта кулоновского взаимодействия. Отсюда мы заключаем, что должна возникнуть локализованная в реальном пространстве макроскопическая структура для любого фиксированного значения ν из ν<1. Так, к примеру, поскольку фактор заполнения уровня Ландау (ФЗУЛ) для квазичастиц дробного квантового эффекта Холла (ДКЭХ) равняется энионному параметру, то эти квазичастицы должны конденсироваться в некоторую структуру в реальном пространстве при данном ФЗУЛ, формируя плато ДЭКХ и показывая, что ДЭКХ является результатом энионных корреляций. С другой стороны, выражая упомянутую выше функцию минимумов по ν через параметр плотности r_s кулоновского газа, мы получаем значения r_s, при которых можно наблюдать 2D вигнеровский кристалл (ВК). Интересно отметить, что одновременное наблюдение в эксперименте ДЭКХ и ВК состояний находится в соответствии с нашим рассмотрением.

Ключевые слова: энион, корреляция, 2D заряженный газ, макроскопическая структура.

Recently, we have derived an approximate analytic formula for the ground state energy of (2D) charged anyon gas. This result allows us in the present paper to estimate the exchange-correlation energy, which is as it is well known from a literature the difference of the ground state energy and the Hartree-Fock energy, for fermions with the anyon parameter ν=1, and to formulate the anyon correlation energy for other ν<1. The minima of the last one for all fixed values of ν as well as the minima of the ground state energy at the same ν are only the ν dependent, which means they are only the anyon statistics dependent with no effect of the Coulomb interaction. From this, we conclude that there should appear the localized real space macroscopic structure for any fixed ν from ν<1. For instance, since the Landau level filling factor (LLFF) of the fractional quantum Hall effect (FQHE) quasi-particles is equal to the

anyon parameter then these quasi-particles should condense in the some real space structure at this LLFF, forming the FQHE plateau and showing that the FQHE is a result of anyon correlations. On the other hand, expressing the above mentioned minima function of ν through the density parameter r_s of the Coulomb gas, we obtain the values of r_s, for which the 2D Wigner crystal (WC) can be observed. It is interesting that the simultaneous observation of the FQHE and WC states in accord with our consideration.

Keywords: anyon, correlation, 2D charged gas, macroscopic structure

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The two-dimensional (2D) gas of electrons or holes of semiconductors, measured at strong external magnetic fields and low temperatures, displays very interesting phenomenon, which calls the fractional quantum Hall effect (FQHE). In this effect *xx* and *xy* components of the semiconductor magneto-resistance exhibit deep minima and plateaus, respectively, at the fixed fractional numerical values of the Landau level filling factor (LLFF) [1]. Three physical factors: inter-particle Coulomb interaction, the strong magnetic field and an external disorder, are crucial for the formation of the FQHE [2, 3]. It was suggested in the reviews [2, 3] that above mentioned plateaus of the FQHE are results of a disorder and impurities in a sample. The effect of localization of the FQHE real space structure as well as of a 2D Wigner crystal (WC) in this disorder was demonstrated in the Ref. [4].

However, as it was demonstrated in the experimental paper, Ref. [1], and in the theoretical work, Ref. [5], these structures are free in the space and not pinned by the disorder and impurities. In the present paper we show that the localization of the FQHE real space structure and of the WC without the disorder and impurities is possible. For the explanation of this effect, we use the finding (see reviews [2, 3] and book [6]) that elementary excitations or quasi-particles, appearing from the system ground state at some fixed fractional values of the LLFF, are responsible for the FQHE, these quasi-particles are anyons, and the anyon factor equals to the LLFF fractional value. More clearly, we demonstrate in the present paper that the ground state energy of 2D Coulomb interacting anyons without magnetic field has a minimum at any fractional value of the LLFF and the physical origin of the FQHE and the WC is in this minimum, while the physical origin of the minimum is in the correlations of these anyons.

Earlier, there were made several attempts to find the FQHE states through the calculation of the ground state energy minima of 2D gas of electrons in the strong magnetic fields at fixed fractional values of the LLFF [7, 8]. In Ref. [8] for the calculation of minima of the same problem, we have used anyons to represent fermions and bosons. Calculations [7, 8] have demonstrated that investigated energy had no minima. While, as it will be shown below, the ground state energy of the FQHE quasi-particles has minimum at any fixed fractional value of the anyon factor or LLFF and this minimum is a result of charged anyon correlations and not of localization and pinning in the disorder or in impurities.

At beginning, we define the correlations of 2D Coulomb interacting anyons. However first, we would like to remember how the correlations for Coulomb interacting electrons are introduced.

The homogeneous electron gas, although it has become the testing ground of quantum mechanical many-body techniques since long time and a result of such calculations can be found in textbooks (see for example [9] and [10]), does still contain yet unexpected puzzles. They are related to the correlation effects caused by the particle-particle interaction, which can be treated so far (or principally) only in approximate ways. Here we would like to draw attention to the 2D systems, which have their prominent realizations in semiconductor heterostructures and layered metallic systems such as the cuprate superconductors. These systems attract much interest from experiment and theory due to the physics related with correlation effects [11-13].

With respect to the ground state energy in fermion systems, the Hartree-Fock (HF) approximation, originally developed for high particle densities, provides the dominating contributions even for intermediate values of the density parameter r_s. Here, r_s is defined as the ratio of the mean distance between particles to the Bohr radius. The difference between the exact ground state energy and its HF value is known as the exchange-correlation energy (see, for example, textbooks [9-11]). For 2D spin-polarized fermions (in this paper we consider only these particles as fer-

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mions), it has been demonstrated by quantum Monte-Carlo (MC) calculations [14, 15] that the exchange-correlation energy may be considered essentially as a small correction to the HF energy for all values of r_s. To find the exact ground state energy of 2D Coulomb interacting boson system, the MC method [16] was also used.

Typically, all MC methods calculate ground state energies for fixed and discrete values of r_s and then an interpolation of these energies is made to yield their expansion in terms of powers of r_s. For bosons, at small r_s (the high density limit), this interpolation formula reproduces the ground state energy of the 2D Coulomb boson gas (2DCBG) obtained with the hypernetted-chain approximation [17]. The latter result follows also by using the Bogoliubov approximation [18]. For fermions, as we said above, the interpolation formula reproduces the ground state energy close to the HF approximation.

For charged anyon systems, correlations between particles have not a simple relationship with respect to exchange of two particles in the real space as for fermions (see, for example, Refs. [11-13]) due to the fact that the exchange of two particles for anyons involves the coordinates of all particles [6] of system. Therefore, it is better do not call the correlations for charged anyons as the exchange-correlations as for fermions. In this case, we call these physical phenomena as anyon correlations, or just correlations. The physics of these correlations will be clarified below.

Here, we outline the main result of Ref. [19], which allows us to conclude about possibility to appear of FQHE and WC states in the charged anyon system without disorder and impurities. It is in the minimum of the ground state energy in the 2D Coulomb interacting gases at each fixed anyon parameter or LLFF.

In Ref. [19] we have employed the anyon concept in order to find an approximate analytic formula (AAF) for the ground state energy of 2D Coulomb gases per particle, which is capable of accounting for the particle fractional anyon statistics together with bosonic and fermionic limits of this statistics. This formula was derived by using the corresponding result for the harmonically confined 2D Coulomb anyon gas [18] and applying a regularization procedure for vanishing of this confinement. After the regularization procedure our AAF becomes the function of the fractional statistics ν and the Coulomb interaction density r_s parameter, i.e. $\varepsilon_0(\nu,r_s)$. For the fermion limit of anyons (with $\nu=1$) and at very small r_s (the high density limit), $\varepsilon_0(\nu,r_s)$ reproduces the HF energy of spin-polarized 2D electrons (Ref. [20])

$$E_{\mathrm{HF}} = \frac{2}{r_s^2} - \frac{16}{3\pi r_s}. \qquad (1)$$

For the boson limit of anyons (with $\nu = 0$) and at the same high density very small r_s values, $\varepsilon_0(\nu,r_s)$ reproduces the ground state energy of the 2D Coulomb boson gas, 2DCBG, found in Refs. [17, 18] (see above). For very large values of r_s, the ground state energy per particle is insensitive to the statistics of particles and $\varepsilon_0(\nu,r_s)$, derived in Ref. [19], reproduces the energy of the classical 2D electron crystal (the 2D WC [21]).

However, the main property of the function $\varepsilon_0(\nu,r_s)$ is in that it has the fixed value of minimum $\varepsilon_{0,\min}$, the position of which is a function of r_s (at constant ν) and determined by fitting procedure just outlined in the previous paragraph.

For the comparison, we depict in Fig. 1 the exchange-correlation energy as the difference of the ground state energies obtained from the MC calculations [14, 15] and HF values (from Eq. (1)) and the correlation energy as the difference of the ground state energies obtained from the expression $\varepsilon_0(\nu=1,r_s)$ and HF values (from Eq. (1)) for 2D fermions.

Figure 1 shows both correlation energies as a function of r_s and exhibits a pronounced minimum for the correlation energy $\varepsilon_{c,\min}(r_s\cong 0.2)\sim -0.8$ Ry, where Ry is Rydberg energy unit. The origin of this property of $\varepsilon_{c,\min}(r_s)$ is in the two facts (see Ref. [18]). First of all, the position of the minimum ($\varepsilon_{0,\min}$) of the energy $\varepsilon_0(\nu=1,r_s)$ as function of r_s ($r_s\approx 0.7$) is shifted on the left with respect to the one of HF ($r_s\approx 2.36$). And at the second, the value of the former ($\varepsilon_{0,\min}=-1.159$) is lower than the one of the later ($E_{\mathrm{HF,min}} = -0.36$).

The another remarkable property of the ground state energy, $\varepsilon_0(\nu,r_s)$, of the charged anyon gas, obtained from AAF, is in that at all $\nu\le r_s$ and $r_s\le 0.5$ it exhibits the prominent minimum $\varepsilon_{0,\min}(\nu\neq 0, r_s)$ (see the Fig. 2). Furthermore, using the approximate expression

$$\varepsilon_0(\nu,r_s) \approx \frac{c_{\mathrm{WC}}^{2/3} f^{2/3}(\nu,r_s)}{r_s^{4/3}} \left(-1 + \frac{7\nu r_s f^{2/3}(\nu,r_s)}{3c_{\mathrm{WC}}^{4/3} r 4_s^{4/3})} \right) \qquad (2)$$

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valid for $\nu \le r_s$ and $r_s \to 0$ and for $\nu=1$ and $r_s \to \infty$ limits, one can show that the $\varepsilon_{0,\min}(\nu \neq 0, r_s)$ is only the function of the anyon parameter ν. For this, we minimize Eq. (2) with respect to $f^{2/3}(\nu, r_s)/r_s^{4/3}$ and obtain the expression $\varepsilon_{0,\min}(\nu) = -3c_{WC}^2/(28\nu)$ where c_{WC}= 3.2903 and it should be $\nu \neq 0$.

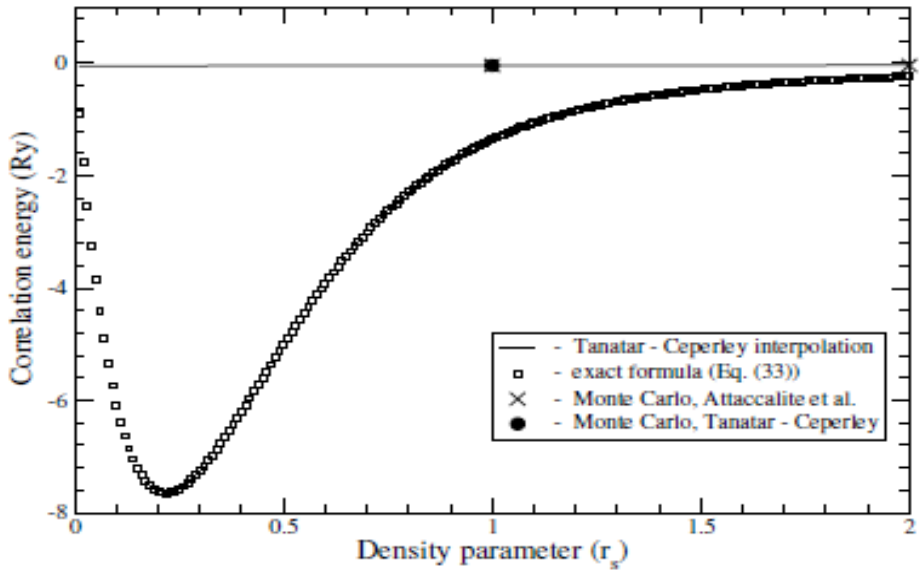


Figure 1. The exchange-correlation energy as the difference of the ground state energies obtained from the MC calculations [14, 15] and HF values (from Eq. (1)) (cross and solid dots, respectively, and their interpolation from Ref. [14] (solid line)) and the correlation energy as the difference of the ground state energies obtained from the expression $\varepsilon_0(\nu=1, r_s)$ and HF values (from Eq. (1)) (open squares) for 2D fermions. In the insert, it is indicated the accurate expression of $\varepsilon_0(\nu, r_s)$ for all values of ν and r_s as "exact formula Eq. (33)" from Ref. [18].

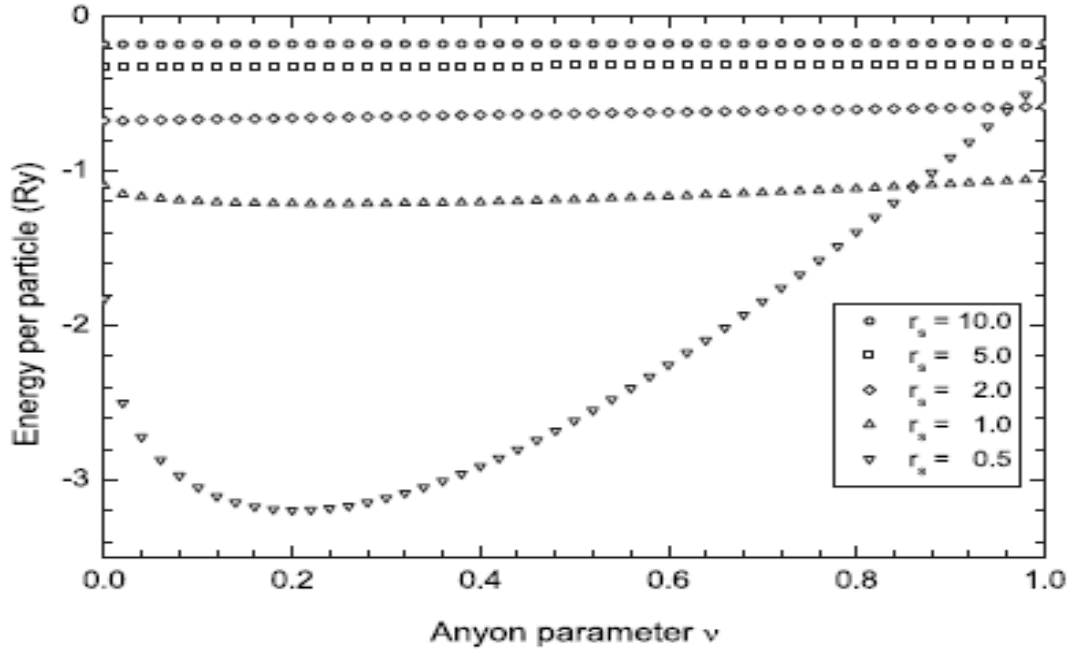


Figure 2. Ground state energies per particle vs anyon parameter ν for various fixed values of r_s. The figure is taken from Ref. [18].

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For the fermion system (ν = 1) this gives the energy minimum $\varepsilon_{0,\min}(\nu)=-1.159$, which exactly coincides with $\varepsilon_0(\nu=1,r_s\approx0.7)$ (see previous paragraph above and Fig. 2). The curves, depicted in Fig. 2, were calculated using AAF for the ground state energy per particle $\varepsilon_0(\nu,r_s)$, Eq. (33) from the Ref. [18], accurate for all values of ν and r_s. A smooth variation of minimal energies $\varepsilon_{0,\min}(\nu)$ with the decrease of the parameter ν is clearly seen from these curves. This property of the $\varepsilon_{0,\min}(\nu)$ shows that it is inherent for the physics of the charged anyon gas and not an artifact of the theory. On the other hand, the ν parameter dependent minimal energy $\varepsilon_{0,\min}(\nu)$ at ν<1 may be universal for anyon gases with arbitrary interparticle interaction.

There appears the question how to define the anyon correlation energy, if HF energy, Eq. (1), is only the density parameter r_s dependent. The Fig. 2 shows the way for that. Physically, it is clear that the correlation energy should display the changing of energy of an anyon system at the variation of the statistics of particles. Therefore, as HF energy in our case, we can bring the energy $\varepsilon_0(\nu,r_s=1.0)$ = const (see Fig. 2), and the correlation energy can be defined as $\varepsilon_0(\nu,r_s<1.0)-\varepsilon_0(\nu,r_s=1.0)$. Since for all fixed values $r_s\geq1.0$ the function $\varepsilon_0(\nu,r_s\geq1.0)$= const (see the same figure).

In principle, we can bring the asymptotic expression for the ground state energy (see Eq. (38) of the Ref. [18])

$$\varepsilon_0(\nu,r_s\to0)\approx\frac{2f(\nu,r_s)}{r_s^2}\left(\nu^{1/2}-\frac{c_{\mathrm{WC}}r_s}{\nu^{1/4}f^{1/2}(\nu,r_s)}\right) \quad (3)$$

valid for $r_s\to0$ and $\nu\neq0$ with the approximate $f(\nu,r_s)\approx\nu^{1/2}$ for the definition of the HF energy of a charged anyon gas. However, for fermions (ν=1), Eq. (3) gives for the minimum energy $\varepsilon_{0,\min}(\nu=1,r_s\to0)=c_{WC}^2/2$, which is lower than $\varepsilon_{0,\min}(\nu=1)=-3c_{WC}^2/28$ (see above), therefore, it is not applicable here as definition of the HF energy. Because one assumes (see, for example, Refs. [14, 15]) that the true energy should be lower than the HF energy.

The anyon parameter ν dependent minimal energy $\varepsilon_{0,\min}(\nu)$ (at ν<1) gives us the unique opportunity to use it for the possible interpretation of experiments on the FQHE plateaus for fractional values of the LLFF. Indeed, the minimum of the ground state energy $\varepsilon_0(\nu,r_s)$ lower than for the boson ground state energy (see Fig. 2), presumes the transition of a system into this state with formation of the real space localized macroscopic structure. It is proven in Refs. [2, 3, 6] that the LLFF of the FQHE plateau is equal to the anyon parameter ν. Therefore, one can expect the existence of the above structure, which forms the FQHE plateau in the electric resistivity or conductivity, for each fixed value of LLFF or ν<1. An analysis, given just above in this paper, gives us a background to consider that the FQHE is a result of (anyon) correlations and not of the Coulomb interaction. Furthermore, one can assume that the FQHE state may be observed in any system with anyon statistics.

The minimal energy $\varepsilon_{0,\min}(\nu)$ (at ν<1) can be expressed as function of the density parameter r_s of the 2D Coulomb gas. This formula can be derived minimizing Eq. (2) for the $\varepsilon_0(\nu,r_s)$ with respect to r_s. To calculate this we use the approximate $f(\nu,r_s\to0)\approx\nu^{1/2}b_1$. In Ref. [18], we motivate the inclusion of the $\nu^{1/2}$ dependence in $f(\nu,r_s)$ by the linear ν dependence of the ground state energy for the anyon gas without Coulomb interaction and close to bosonic limit (ν=0) found in [22] and [23]. This linear dependence of the energy on ν is also reproduced in Eq. (3).

A minimization of Eq. (2) with $f(\nu,r_s\to0)\approx\nu^{1/2}b_1$ over the r_s gives (see also Ref. [18]) $\varepsilon_{0,\min}=-(4/5)b_2^{1/4}c_{WC}b_1^{1/2}/r_s$, which occurs at position $\nu_0=b_2^{3/4}c_{WC}r_s/b_1^{1/2}$, where b_1=1 and b_2=3/35. For r_s=0.2 this provides $\varepsilon_{0,\min}=-8.689$ and $\nu_0\approx0.08$, while from AAF expression $\varepsilon_0(\nu,r_s)$ we have $\varepsilon_{0,\min}=-8.847$ and $\nu_0\approx0.08$. Comparing the asymptotic formula $\varepsilon_{0,\min}\sim1/r_s$ with the ground state energy expression $\varepsilon_0(\nu=0,r_s\to0)=-c_{BG}r_s^{-2/3}$ of the 2DCBG, where $c_{BG}=1.29355$, derived in Refs. [17, 18], it is evident that at $r_s\to0$ the charged anyon gas ground state has lower energy than one of the charged bosons.

If we look on Fig. 2, we see the appearance of minimums $\varepsilon_{0,\min}$ for each fixed density r_s from $r_s\leq0.5$ range. This takes place since mentioned density r_s corresponds to the fixed anyon parameter ν_0, at which $\varepsilon_{0,\min}(\nu)$ occurs. The idea, used for the observation of the FQHE plateaus within the present treatment, can be applied for the observation of the WC also. This occurs because the localized macroscopic structure of charged particles in the form of the WC may be condensed in the $\varepsilon_{0,\min}$ minimum of the energy. It is

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interesting, the simultaneous coexistence of FQHE and WC states was observed in Ref. [4].

One can estimate the applicability of our treatment for the case of the experiment [4]. The experiment observed the FQHE plateau and WC structures at $r_s \approx 30$. This paper defines the parameter r_s as $r_s = m^* e^2 / (4\pi\delta\hbar^2\sqrt{\pi\rho})$, where m^* and e are the effective mass and charge of a hole, respectively, δ is the dielectric constant of a sample, and ρ is the density of holes. In our consideration, all parameters in the definition of r_s are unchanged except the charge of a hole e. As we said above, the FQHE is the system with a charge νe [6], i.e. the system of FQHE quasi-particles. Therefore, in fact, the parameter r_s should be redefined with the factor ν^2. Thus instead of $r_s \approx 30$ in experiment, we need to say about $r_s \approx \nu^2 30$ and to have a value of r_s from the range $r_s < 1.0$ there should be the value $\nu \approx 0.2$. In the Fig. 2, the case with $\nu \approx 0.2$ corresponds to the minimum of the ground state energy for $r_s = 0.5$. Therefore, in fact, the experiment, Ref. [4], sees $r_s = 0.5$.

In conclusion, in the frame of our phenomenological approach based on the anyon concept, we have identified exchange-correlation and anyon correlation effects in the high density limit of the 2D Coulomb anyon gas. In contrast with results from MC calculations for fermions, our data for the charged anyon gas show a pronounced minimum in the ground state and correlation energies, which were ascribed to the anyon correlations. We used this result with minimum of both energies for the possible observation of FQHE and WC states, in which the role of anyon statistics is crucial, and find agreement with the experiment.

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Икки-ўлчамли кулон газлар ичида энион корреляциалар натижасида ҳосил бўладиган фракцияли Холл квант эффекти ва Вигнер кристалл ҳолатларнинг мумкинлиги

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Биз яқинда чоп этилган мақоламизда, (2D) зарядланган эниoнлар гази учун асосий ҳолат энергиясининг аналитик аппроксимация формуласини чиқара олдик. Бу мақоламизда биз олинган натижаларни ишлатиб, энион параметри ν=1 чегарасида ифода бўладиган фермионлар учун алмашиш-корреляцион энергиясини баҳолашга муясар бўлдик ва қолган ν<1 параметрлар учун энион-корреляцион энергиясини аниқлаш йулини ифодаладик. Алмашиш-корреляцион энергияси асосий ҳолат энергиясидан Хартри-Фок энергиясини фарқига тенглиги адабиетда яхши езиб этилган. Хамма фиксация қилинган ν да энион-корреляцион энергия-сининг минимумлари асосий ҳолат энергиясини минимумларига мос бўлиб фақат ν лар функция-силигини аниқлади. Демак бу минимумлар фақат эниoнлар статистикаси билан боғлиқ бўлиб кулон таъсири эффектидан ироқ экан. Бу аргументлар қуйидаги хулосага келтиради. ν < 1 соҳасидан олинган ихтиерий фиксация қилинган ν миқдорида реал фазасида макроскопик структура ҳосил бўлиши лозим. Мисол учун қасрли Холл квант эффекти (ҚХКЭ) квазизарраларнинг Ландау сатқининг тўлдириш фактори (ЛСТФ) энион параметрига тенглиги сабабли берилган ЛСТФ миқдорида ҚХКЭнинг платосини ҳосил қилиб, бу квазизарралар реал фазасида қандайдир структурасига конденсатланиши лозим. Бу мулохоза ҚХКЭ ҳолати анионлар корреляцияси натижасида пайдо бўлишини кўрсатади. Бошқа тарафдан қаралганда юқорида эслатган ν минимумлар функциясини кулон газнинг зичлик параметри r_s орқали ифодаласак 2D Вигнер кристаллни (ВК) экспериментда кузатиладиган r_s зичликни аниқласа бўлади. Шуни айтиб ўтиш қизиқарлики, бир вақтда экспериментда ҚХКЭ ва ВК лар ҳолатларини кузатилиши бизнинг назариямиз билан мос.

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