Effective triaxiality of even-even nuclei with quadrupole and octupole deformations

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Развита неадиабатическая коллективная модель тяжелых неаксиальных четно-четных ядер с квадрупольной и октупольной деформациями. Поперечные квадрупольные и октупольные колебания поверхности ядра учитываются эффективно, а продольные колебания учитываются динамически. На основе предложенной модели получено аналитическое выражение для энергии уровней коллективных состояний, содержащее пять подгоночных параметров. Эти параметры определяются из сравнения теоретических и экспериментальных данных. Угловая часть полярных координат использована в качестве параметра для равновесного положения четно-четного ядра. Получено хорошее описание коллективных состояний yrast-полос переменной четности тяжелых неаксиальных четно-четных ядер: ^{150}Nd, ^{152,154}Sm, ^{172}Yb, ^{228,230,232}Th, ^{232,234,236,238}U и ^{240}Pu.

Ключевые слова: неадиабатическая коллективная модель, квадрупольные и октупольные деформации, неаксиальность, yrast-полоса.

A nonadiabatic collective model of nonaxial even-even nuclei with quadrupole and octupole deformations is developed. Transverse quadrupole and octupole deformations are taken into account effectively. And longitudinal deformations are taken into account dynamically. On the basis of the proposed model, an analytical expression for the level energy is obtained containing five adjustable parameters, which are determined from a comparison of theoretical and experimental data. The angular part of the polar coordinates is used as a parameter of an equilibrium state of an even-even nucleus. A good description of the collective states of the yrast-band with alternating parity of heavy nonaxial even-even nuclei ^{150}Nd, ^{152,154}Sm, ^{172}Yb, ^{228,230,232}Th, ^{232,234,236,238}U and ^{240}Pu is obtained.

Keywords: nonadiabatic collective model, quadrupole and octupole deformations, nonaxiality, yrast-band.

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I. Introduction

Most of the atomic nuclei are deformed in the ground state. The charactheristics of most deformed nuclei have revealed that they have axially symmetric prolate or oblate shapes. However, atomic nuclei have a stronger tendency to deform into prolate shapes than into oblate ones [1, 2]. In Ref. [3] a systematic study of even-even nuclei was performed within relativistic mean field theory with axial deformation. The ground state properties of about 2000 even-even nuclei from Z = 8 to Z = 120 have been studied and all possible deformations of each nuclide have been examined. The possible appearance of deformation beyond the axial kind, such as triaxial or even higher order multipole deformations have been suggested.

The fundamental properties of the nuclear shape are expressed by the parameter quadrupole deforma-

tion β_2 and asymmetry parameter γ [4]. The parameter β_2 is related to the intrinsic quadrupole moment and describes the magnitude of departure from the spherical shape along given axis. At $\gamma = 0$ and $\gamma=\pi/3$ it corresponds to the axially symmetric prolate and oblate shapes, respectively. The intermediate values of γ, within $0<\gamma<\pi/3$, give the triaxial shapes [4-6].

The collective spectra of atomic nuclei with axial symmetry quadrupole and octupole deformations ($\gamma=0$, $\eta=0$, and $K=0$, and β_2, β_3 are free dynamical variables) are characterized by rotational bands with alternating parity [7-14]. In Ref. [14] the yrast- and non-yrast bands with alternating parity of deformed axial even-even nuclei within a collective model with nonadiabatically coupled quadrupole and octupole degrees of freedom were described. The structure of the yrast- and first non-yrast alternating parity sequences in the rare-earth nuclei ^{150}Nd, ^{152,154}Sm, ^{154}Gd, ^{156}Dy, ^{162,164}Er and the actinides ^{232,234,236,238}U were reproduced satisfactorily. Where the hamiltonian of the axial-symmetry even-even nuclei has an exact $I(I+1)$ dependence and the observed or estimated deviations from such a simple energy dependence is actual task. It should be noted that in the experiments [15] one can observe energy bands, which cannot be explained within the framework of the nuclei models with axially symmetric multipole deformations [8-14], for example, the spectrum of γ-band energy levels [16, 17].

In Ref. [18] the approach triaxial rotator model (in the case K-mixing, K are the projections of the angular momentum on the third axes of the intrinsic frames) was used for description energy levels with alternating parity of even-even nuclei: ^{228,230,232}Th, ^{230,232,234,236,238}U and ^{240}Pu with triaxial quadrupole and octupole deformations (γ-, η-deformations), in the region actinides. In this consideration the deformation parameters are constants whose values $\beta_{2\text{eff}}$, γ_{eff}, $\beta_{3\text{eff}}$ and η_{eff} effectively determine a rigid triaxial rotor. Therefore, the purpose of the present work is to develop the model of atomic nuclei with triaxial-quadrupole and octupole deformations, i.e. β_2, β_3 are free dynamical variables, and $\gamma=\gamma_{\text{eff}}$, $\eta=\eta_{\text{eff}}$ and $K=0$. This model allows one to describe other energy bands observed in experiments with heavy ions [15]. As it will be seen below the implemented analysis allows us to make a conclusion about the relevance of the applied model formalism.

II. Model of axial quadrupole-octupole vibrations and triaxial rotations

The general theory of quadrupole and octupole excitation of even-even nuclei is determined by the Hamilton operator, which contains seven dynamical variables: deformation parameter β_2 ($\beta_2\geq0$) and asymmetry parameter γ ($0\leq\gamma\leq\pi/3$) of the quadrupole oscillations; deformation parameter β_3 ($\beta_3\geq0$) and asymmetry parameter η ($0\leq\eta\leq\pi/2$) of the octupole oscillations; Eulerian angles θ_1 ($0\leq\theta_1\leq2\pi$), θ_2 ($0\leq\theta_2\leq\pi$), θ_3 ($0\leq\theta_3\leq2\pi$) are determine rotation of the nucleus relative to the laboratory system [18]:

$$\hat{H} = \hat{T}_{\beta_2} + \hat{T}_{\beta_3} + \hat{T}_{\gamma} + \hat{T}_{\eta} + T_{rot} + V(\beta_2,\beta_3,\gamma,\eta), \quad (1)$$

where

$$\hat{T}_{\beta_2} = -\frac{\hbar^2}{2B_2}\frac{1}{\beta_2^4}\frac{\partial}{\partial\beta_2}\left(\beta_2^4\frac{\partial}{\partial\beta_2}\right), \quad (2)$$

$$\hat{T}_{\beta_3} = -\frac{\hbar^2}{2B_3}\frac{1}{\beta_3^4}\frac{\partial}{\partial\beta_3}\left(\beta_3^4\frac{\partial}{\partial\beta_3}\right), \quad (3)$$

$$\hat{T}_{\gamma} = -\frac{\hbar^2}{2B_2}\frac{1}{\beta_2^2\sin(3\gamma)}\frac{\partial}{\partial\gamma}\left(\sin(3\gamma)\frac{\partial}{\partial\gamma}\right), \quad (4)$$

$$\hat{T}_{\eta} = -\frac{\hbar^2}{2B_3}\frac{1}{\beta_3^2\sin(3\eta)}\frac{\partial}{\partial\eta}\left(\sin(3\eta)\frac{\partial}{\partial\eta}\right). \quad (5)$$

Expressions (2), (3), (4) and (5) are kinetic energy of operators β_2, β_3, γ and η vibrations, respectively; $V(\beta_2, \beta_3, \gamma, \eta)$ – potential energy of above mentioned vibrations and

$$\hat{T}_{rot} = \sum_{\lambda=1}^{3}\frac{\hbar^2\hat{I}_i^2}{J_i} \quad (6)$$

is the rotational energy operator, here I_i (i=1,2,3) are the components of the angular momentum operator. J_i are the components of the quadrupole and octupole moment of inertia. In the general case, they depend on the parameters of the mass and the quadrupole and octupole deformations B_2, β_2, γ and B_3, β_3, η, respectively. General solution of Schrodinger equation with hamiltonian (1) is a very complicated. Therefore different simplifications are used. One such model is the analog of Davydov-Chaban model [16, 17], where variables γ and η are replaced by their effective values γ_{eff} and η_{eff}. Then, the hamiltonian (1) takes the following form:

$$\hat{H}_{total} = \hat{T}_{\beta_2} + \hat{T}_{\beta_3} + \hat{T}_{rot} + V(\beta_2,\beta_3), \quad (7)$$

where $\hat{T}_{\beta_2}$ and $\hat{T}_{\beta_3}$ are determined by equations (2) and (3).

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In this case rotational energy operator (6) depends on the effective values γ_{eff} and η_{eff}.

Now we will write Schrodinger equation with hamiltonian (7):

$$-\frac{\hbar^2}{2B_2}\left[\frac{3}{\beta_2}\frac{\partial}{\partial\beta_2}+\frac{\partial^2}{\partial\beta_2^2}\right]\Psi_I^{\pm}(\beta_2,\beta_3,\theta)- \\ -\frac{\hbar^2}{2B_3}\left[\frac{3}{\beta_3}\frac{\partial}{\partial\beta_3}+\frac{\partial^2}{\partial\beta_3^2}\right]\Psi_I^{\pm}(\beta_2,\beta_3,\theta)+\left[\hat{T}_{rot}+\cdot\right. \\ \left.+V(\beta_2,\beta_3)\Psi_I^{\pm}(\beta_2,\beta_3,\theta)\right]=E_I^{\pm}\Psi_I^{\pm}(\beta_2,\beta_3,\theta). \quad (8)$$

We go to polar coordinates σ ($0\le\sigma\le\infty$) and ε ($-\pi/2\le\varepsilon\le\pi/2$).

$$\beta_2=\sqrt{\frac{B}{B_2}}\sigma\cos\varepsilon,\ \beta_3=\sqrt{\frac{B}{B_3}}\sigma\sin\varepsilon,\ B=\frac{B_2+B_3}{2}. \quad (9)$$

$$-\frac{\hbar^2}{2B}\left[\frac{\mathrm{d}^2}{\mathrm{d}\sigma^2}+\frac{1}{\sigma}\frac{\mathrm{d}}{\mathrm{d}\sigma}+\frac{1}{\sigma^2}\frac{\mathrm{d}^2}{\mathrm{d}\varepsilon^2}\right]+ \\ +\left[\hat{T}_{rot}+V(\sigma,\varepsilon)-\xi^{\pm}\right]\Psi_I^{\pm}(\sigma,\varepsilon,\theta)=0. \quad (10)$$

The solution to equation (9) is very difficult. Therefore, further we assume, that

$$\hat{T}_{rot}=\frac{\hbar^2}{2B\sigma^2}\hat{H}_a. \quad (11)$$

Here

$$\hat{H}_a=I_1^2/J_1^2+I_2^2/J_2^2+I_3^2/J_3^2, \quad (12)$$

$$J_1=\cos^2(\varepsilon)\sin^2(\gamma_{eff}-\frac{2\pi}{3})+\sin^2(\varepsilon)\left[\frac{3}{2}\cos^2\eta_{eff}+\sin^2\eta_{eff}+\frac{\sqrt{15}}{2}\sin\eta_{eff}\cos\eta_{eff}\right], \quad (13)$$

$$J_2=\cos^2(\varepsilon)\sin^2(\gamma_{eff}-\frac{4\pi}{3})+\sin^2(\varepsilon)\left[\frac{3}{2}\cos^2\eta_{eff}+\sin^2\eta_{eff}+\frac{\sqrt{15}}{2}\sin\eta_{eff}\cos\eta_{eff}\right], \quad (14)$$

$$J_3=\cos^2(\varepsilon)\sin^2(\gamma_{eff})+\sin^2(\varepsilon)\sin^2\eta_{eff}. \quad (15)$$

We then consider the collective rotational motion of the nucleus separately from the other degrees of freedom. If no further assumptions (e.g. axial symmetry) are imposed, the corresponding energy spectrum is described by the triaxial-rotor Hamiltonian (12). Taking into account the expressions (13), (14) and (15) for J_i, where J_i explicitly depend on the effective values of the quadrupole and octupole deformation parameters γ_{eff}, η_{eff} and ε. As a first approximation, we can use the effective value of this variable, i.e. ε_{eff}.

Thus, implying

$$\Psi_I^{\pm}(\sigma,\theta)=F_I^{\pm}(\sigma)\Phi_{I\tau}^{\pm}(\theta), \quad (16)$$

and using rigid rotor approximation, we rewrite Eq. (8) as in Ref. [16]

$$\left[\hat{H}_a-\xi_{I\tau}^{\pm}\right]\Phi_{I\tau}^{\pm}(\theta)=0, \quad (17)$$

where I_τ – energy of triaxial rotor [18-20] (see next section). We will write Schrodinger equation with hamiltomian in polar coordinates and by taking into account $\varepsilon=\varepsilon_{\text{eff}}$

$$\left\{-\frac{\hbar^2}{2B}\left[\frac{\mathrm{d}^2}{\mathrm{d}\sigma^2}+\frac{1}{\sigma}\frac{\mathrm{d}}{\mathrm{d}\sigma}\right]+\left[\frac{\hbar^2\xi_I^{\pm}}{4B\sigma^2}+V(\sigma)-E_I^{\pm}\right]\right\}F_I^{\pm}(\sigma)=0. \quad (18)$$

III. Solution for the triaxial quadrupole-octupole rotor

In the present work we consider the deformation parameters as constants whose values γ_{eff}, η_{eff}, ε_{eff} effectively determine a rigid triaxial rotor [6, 15, 16]. Thus, we obtain the quadrupole-octupole moment of inertia in a form depending on three arguments $J_i = J_i(\gamma_{\text{eff}}, \eta_{\text{eff}}, \varepsilon_{\text{eff}})$. We take these arguments, γ_{eff}, η_{eff}, ε_{eff} as adjustable parameters. By introducing the reciprocal inertia factors $A_i(\gamma_{\text{eff}}, \eta_{\text{eff}}, \varepsilon_{\text{eff}})=1/[2J_i(\gamma_{\text{eff}}, \eta_{\text{eff}}, \varepsilon_{\text{eff}})]$ (i=1,2,3) we get the Hamiltonian (6) in the form

$$\hat{H}_a=\sum_{i=1}^{3}\hbar^2A_i(\gamma_{eff},\eta_{eff},\varepsilon_{eff})\hat{I}_i^2 \quad (19)$$

depending on the three parameters γ_{eff}, η_{eff}, ε_{eff}. The energy spectrum of the above determined triaxial quadrupole-octupole rotor is obtained by diagonalizing the Hamiltonian (6) on the basis of the symmetrized rotor functions with built-in parity [19]

$$\left|IMK,\pm\right>=\sqrt{\frac{1}{2(1+\sigma_{Ko})}}\left\langle IMK\left|\pm(-1)^I\right|IM-K\right\rangle, \quad (20)$$

where $\left|IMK\right>=\sqrt{\frac{2I+1}{8\pi^2}}D_{MK}^{I}(\theta)$ and M and K are the

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projections of the angular momentum $\hat{I}$ on the third axes of the laboratory and intrinsic frames, respectively. The eigenfunctions are obtained in the form

$$\Phi^{\pm}_{IM\tau}(\theta) = \sum_{K\geq 0}^{I} C^{\tau}_{IK} \left| IMK, \pm \right> . \qquad (21)$$

The expansion coefficients implicitly depend on the effective mass and deformation parameters $C^{\tau}_{IK} = C^{\tau}_{IK}(\gamma_{eff}, \eta_{eff}, \varepsilon_{eff})$.

For given angular momentum I the quantum number τ = 0, 1, 2,… (limited by $K \leq I$) labels the different eigenvalues $E_{\tau}(I)$ of $\hat{H}_a$ in ascending order.

We obtain the matrix elements of Hamiltonian (6) between basis states (22) with different K values in the following general form:

$$\begin{aligned}\left\langle IMK' \pm \left| \hat{H}_a \right| IMK \pm \right\rangle = \sqrt{\frac{1}{(1+\delta_{K',0})(1+\delta_{K,0})}} \Big\{ \frac{1}{2} \Big[(A_1 + A_2) I(I+1) - (A_1 + A_2 - 2A_3) K^2 \Big] \times \\ \times \Big[\delta_{K',K} \pm (-1)^{I-K} \delta_{K',-K} \Big] + (A_1 - A_2) f(I,K) \Big[\delta_{K',K+2} \pm (-1)^{I-K} \delta_{K',-\mathrm{K}-2} \Big] + \\ + (A_1 - A_2) f(I,-K) \Big[\delta_{K',K-2} \pm (-1)^{I-K} \delta_{K',-K+2} \Big] \Big\}\end{aligned} \qquad (22)$$

with

$$f(I,K) = \frac{1}{4} \sqrt{(I+K+2)(I+K+1)(I-K-1)(I-K)} . \qquad (23)$$

Since K, $K' \geq 0$ in (22), one has $\delta_{K',-K} \neq 0$ only for $K = 0$, $\delta_{K',-K+2} \neq 0$ only for $K = 0$, 1, 2, while $\delta_{K',-K-2}$ is always zero. As a result Eq. (22) can be written in a simpler form

$$\begin{aligned}\left\langle IMK' \pm \left| \hat{H}_a \right| IMK \pm \right\rangle = \frac{1}{2} \sqrt{\frac{1}{(1+\delta_{K',0})(1+\delta_{K,0})}} \Big\{ \Big[(A_1 + A_2) I(I+1) - (A_1 + A_2 - 2A_3) K^2 \Big] \times \\ \times \Big[\delta_{K',K} \pm (-1)^{I-K} \delta_{K',-K} \Big] + 2(A_1 - A_2) \Big[f(I,K) \delta_{K',K+2} + f(I,-K) \delta_{K',\mathrm{K}-2} \pm (-1)^{I-K} \delta_{K',-K+2} \Big] \Big\}\end{aligned} . \qquad (24)$$

Here we make the following important assumptions originally imposed in [13[:

(i) The quantum number K takes only even values (see also [12, 20]). Then in (21) the sum over K runs from 0 to I in steps of 2.

(ii) The (+) and (−) phases in (21) and (20), which correspond to the A and B_1 classes of the rotation group D_2, are attributed to the positive and negative parity states, respectively.

Because of (i), the phase factor $(-1)^K = 1$ can be omitted in the matrix element expressions (22) and (24). According to (ii) one can write $\Phi^{\pm}_{IM\tau}(\theta) \equiv \Phi^{\pi}_{IM\tau}(\theta)$ and $\left| IMK, \pm \right> \equiv \left| IMK\pi \right> \equiv$ in (21) and (20), respectively, where $\pi = \pm$ is the parity of the rotation state. Also, according to (20) the $\pi = (+)$ parity corresponds to I = even angular momentum states and $\pi = (-)$ corresponds to I = odd ones, as in the usual alternating-parity (octupole) bands. Therefore, one has $\pi = (-1)^I$. Then the (±) and phase factors in (22) and (24) can be omitted when alternating-parity bands are considered. As a result the matrix element (24) reads as

$$\begin{aligned}\left\langle IMK'\pi \left| \hat{H}_a \right| IMK\pi \right\rangle = \frac{1}{2} \sqrt{\frac{1}{(1+\delta_{K',0})(1+\delta_{K,0})}} \Big\{ \Big[(A_1 + A_2) I(I+1) - (A_1 + A_2 - 2A_3) K^2 \Big] \times \\ \times \Big[\delta_{K',K} + \delta_{K',-K} \Big] + 2(A_1 - A_2) \Big[f(I,K) \delta_{K',K+2} + f(I,-K) \Big[\delta_{K',K-2} + \delta_{K',-K+2} \Big] \Big] \Big\}\end{aligned} . \qquad (25)$$

Note that the right hand side of (25) does not include π explicitly, whereas as mentioned above, it is implied by the value of I (even or odd). The lowest eigenvalues $E^{\tau=0}(I)$ and eigenfunctions $\Phi^{\pi}_{IM(\tau=0)}$ of the Hamiltonian correspond to the ground-state-band for I =even, and to the lowest negative-parity band for I =odd. In the same way the higher eigenfunctions/eigenvalues with τ=1, 2, ... determine higher

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sets of positive- and negative-parity bands. In this work we only consider the lowest (yrast) band structure with $\tau = 0$.

IV. Radial Schrodinger equation

For defined solution (18) potential $V(\sigma)$ can offer

$$V(\sigma) = V_0(\frac{\sigma}{\sigma_0} + \frac{\sigma_0}{\sigma})^2, \quad (26)$$

where V_0 is potential energy in ground state. We bring in designation

$$x = \frac{2BV_0}{\hbar\sigma_0}\sigma^2. \quad (27)$$

Then we obtained following equation

$$\left[\frac{\partial}{\partial x} + \frac{1}{x}\frac{\partial}{\partial x} - \frac{s^2}{x^2} + \frac{E_I^{\pm}}{x}\right]F_I^{\pm}(\sigma) = 0 \quad (28)$$

and taking into account boundary conditions for Schrodinger equations we obtain

$$\left[x\frac{\partial}{\partial x} + (2s - 1 - x)\frac{\partial}{\partial x} + E_I^{\pm} - s - 1/2\right]F_I^{\pm}(x) = 0, \quad (29)$$

$$s = \sqrt{\frac{\epsilon_{I\tau}}{8} + \frac{1}{\mu^4}}$$

where μ is nonadiabatic parameter [16] and

$$\mu^4 = \frac{\hbar^2}{2BV_0\sigma_0^2}.$$

The eigenfunctions

$$F_I^{\pm}(x) = N_\sigma x^8 \exp^8 F(-n, 2s+1, x),$$

N_σ is normalization coefficient, $F(-n, 2s+1, x)$ is confluent hypergeometric function, n = 0, 1, 2, ... are quantum number of σ-vibrations, where for n=0 the energy levels correspond to yrast-band, and for n=1 the energy levels correspond to first non-yrast-band, etc.

The eigenvalues

$$E_I^{\pm} = \left[2n + 1 + \sqrt{\frac{\epsilon_{I\tau}}{8} + \frac{1}{\mu^4}}\right]\frac{V_0}{\mu^2} - 2V_0. \quad (30)$$

Energy levels in ground state:

$$E_{001}^{+} = \left[1 + \frac{1}{\mu^2}\right]\frac{V_0}{\mu^2} - 2V_0.$$

Energy of the excited states is defined by following expression

$$E_{In\tau}^{\pm} = \left[2n + \sqrt{\frac{\epsilon_{I\tau}}{8} + \frac{1}{\mu^4}} - \frac{1}{\mu^2}\right]\frac{V_0}{\mu^2}. \quad (31)$$

Introduced following quantity $\hbar\omega_0 = V_0/\mu^2$ is energy factor [11, 14]. Then Eq. (31) can be written as

$$E_{In\tau}^{\pm} = \left[2n + \sqrt{\frac{\varepsilon_{I\tau}}{8} + \frac{1}{\mu^4}} - \frac{1}{\mu^2}\right]\hbar\omega_0. \quad (32)$$

In the presented approximation, the following fitting parameters are used: $\hbar\omega_0$ (in keV), γ_{eff} (in degrees), η_{eff} (in degrees) and ε_{eff} (in degrees) and μ (dimensionless).

In general, the ranges of triaxiality parameters are $0° < \eta_{eff} < 90°$, $0° < \gamma_{eff} < 60°$ and $-90° < \varepsilon_{eff} < 90°$.

Many approaches exist for calculating the asymmetry parameter γ_{eff} from experimental observables such as $R_{4/2}=E_{4+}/E_{2+}$ and $B(E_2)$. Davydov and Filippov [6] used this ratio to determine γ_{eff}, as well as Varshni and Bose estimated γ_{eff} by using the ratio $R_{4/2}$ [5]. And the calculation of γ_{eff} from $B(E_2)$ values leads to ambiguous values of γ_{eff} and more reliable values are expected from the level energies [5]. But an estimation of asymmetry parameters of quadrupole and octupole deformations η_{eff} and γ_{eff} was not carried out. The same attempt was made in Ref. [22], where the experimental data for negative parity states in ^{228}Th and ^{232,234}U has been compared with the predictions of three possible asymmetric rotor models: pure quadrupole, pure octupole and mixed-quadrupole/octupole.

A simple estimation of the influence of γ-deformation mode can be done by assuming small variations of the system around $\gamma = 0$, as in the case of the $X(5)$ model [23]. In Ref. [9], the possible ways of the influence of the γ-deformation mode on the collective motion can be estimated in the β_2-β_3 space. Further in this work, the possibility of taking into account the triaxiality of the octupole degree of freedom is noted. In present work we effectively take into account triaxiality of the quadrupole and octupole degrees of freedom, i.e. we use quantities γ_{eff} and η_{eff} as adjustable parameters. This is developed of our work for dynamical β_2 and β_3 variables [18], where the possibility description of the yrast positive- and negative-parity excitations of deformed even-even nuclei through a collective rotation model is given in which the nuclear surface is characterized with effective triaxial-quadrupole and octupole deformations.

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V. Results

The obtained values of the parameters: ω_σ, γ_{eff}, η_{eff}, ε_{eff}, μ as well as the root-mean-square (RMS) deviations of the energy levels (in keV) between theory and experiment are given in the captions of Figs. [1-12] for all considered even-even nuclei: ^{154,160}Gd, ^{154}Sm, ^{156}Dy, ^{228,230,232}Th, ^{230,232,236,238}U and ^{240}Pu. In these figures (left plots) are presented a comparison of the theoretical and experimental ener gy levels of the yrast-band. The theoretical results are obtained by (31) with n=0. The behaviour theoretical energy levels of the yrast-band are presented for all nuclei is in a good agreement with experimental data.

In present work fitting parameters ω_σ, γ_{eff}, η_{eff}, ε_{eff}, μ are determined by least squares method.

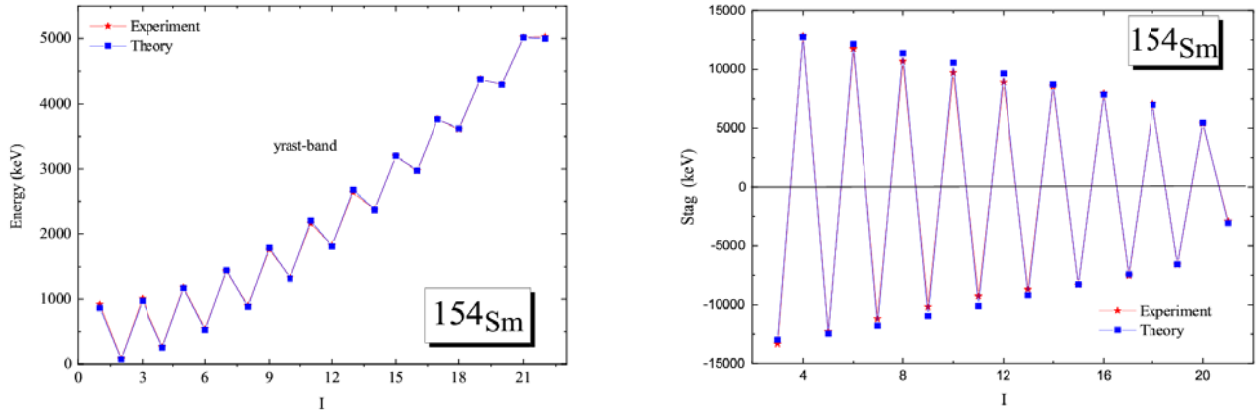


Figure 1. Theoretical and experimental energy levels of yrast-band with alternating parity (left) and "staggering" pattern (right) for ^{154}Gd with parameters: $\hbar\omega_0$=726.01 keV, μ=0.3666, ε_{eff}=0.00040, γ_{eff}=53.930, η_{eff}=85.520, RMS=72.73 keV.

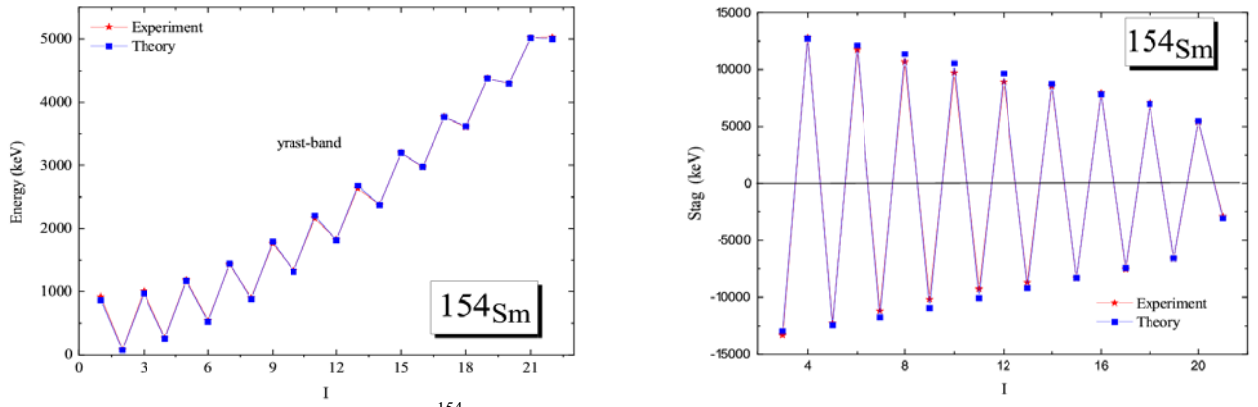


Figure 2. The same as in Fig. 1, but for ^{154}Sm with parameters: ω = 809.65 keV, μ = 0.306, ε_{eff} = 0.00240, γ_{eff} = 54.090, η_{eff} = 84.60, RMS = 22.73 keV.

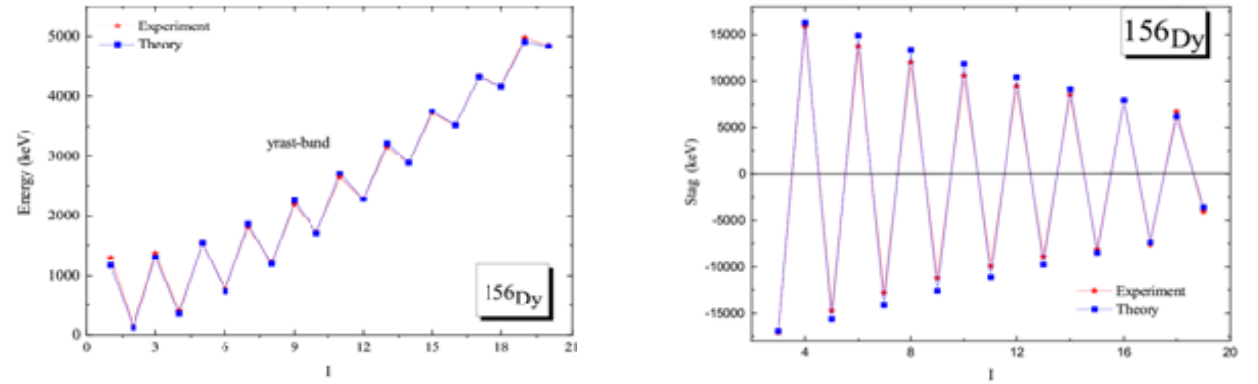


Figure 3. The same as in Fig. 1, but for ^{156}Dy with $\hbar\omega$=649.37 keV, μ=0.4165, ε_{eff}=0.0025°, γ_{eff}=54.09°, η_{eff}=39.96°, RMS=48.04 keV.

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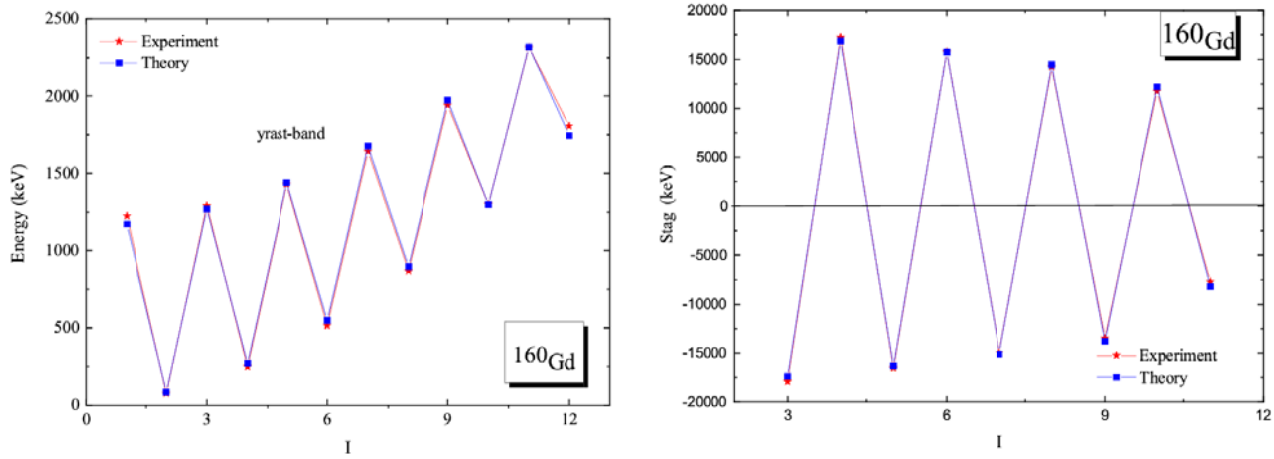


Figure 4. The same as in Fig. 1, but for ^{160}Gd with fitting parameters: $\hbar\omega_0$=531.14 keV, μ=0.3959, ε_{eff}=0.0039°, γ_{eff}=54.98°, η_{eff}=18.93°, RMS=31.81 keV.

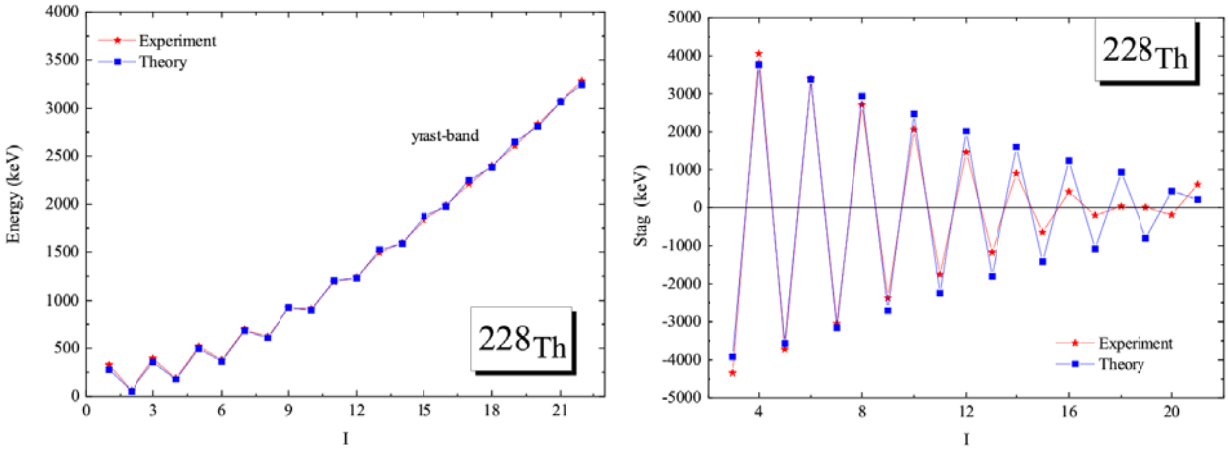


Figure 5. The same as in Fig. 1, but for ^{228}Th with fitting parameters: $\hbar\omega_0$=500.98 keV, μ=0.3217 ε_{eff}=0.0003^0, γ_{eff}=51.26^0, η_{eff}=61.81^0, RMS=24.93 keV.

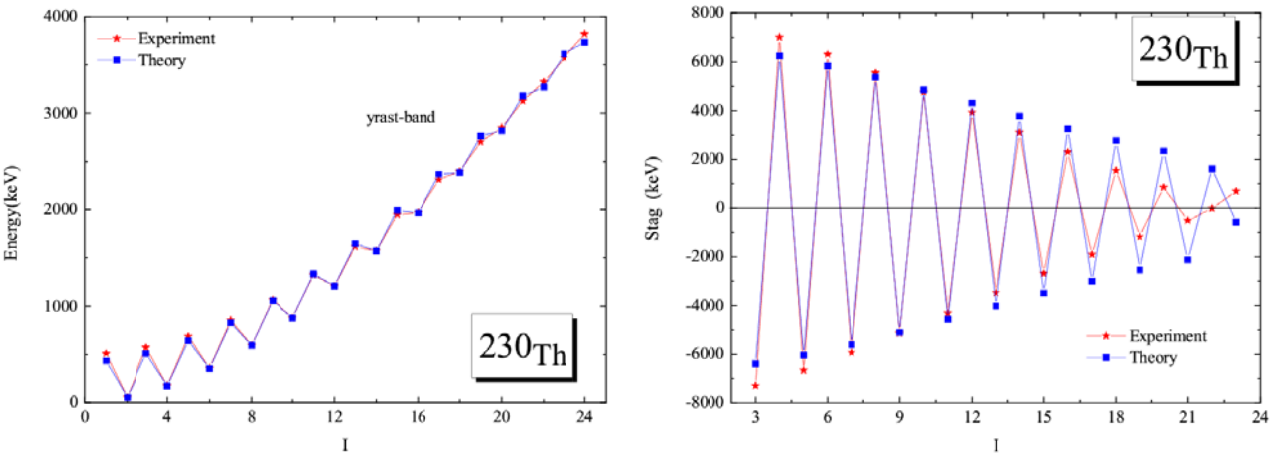


Figure 6. The same as in Fig. 1, but for ^{230}Th with fitting parameters: $\hbar\omega_0$=510.57 keV, μ=0.3155 ε_{eff}=0.0006^0, γ_{eff}=53.14^0, η_{eff}=84.98^0, RMS=40.02 keV.

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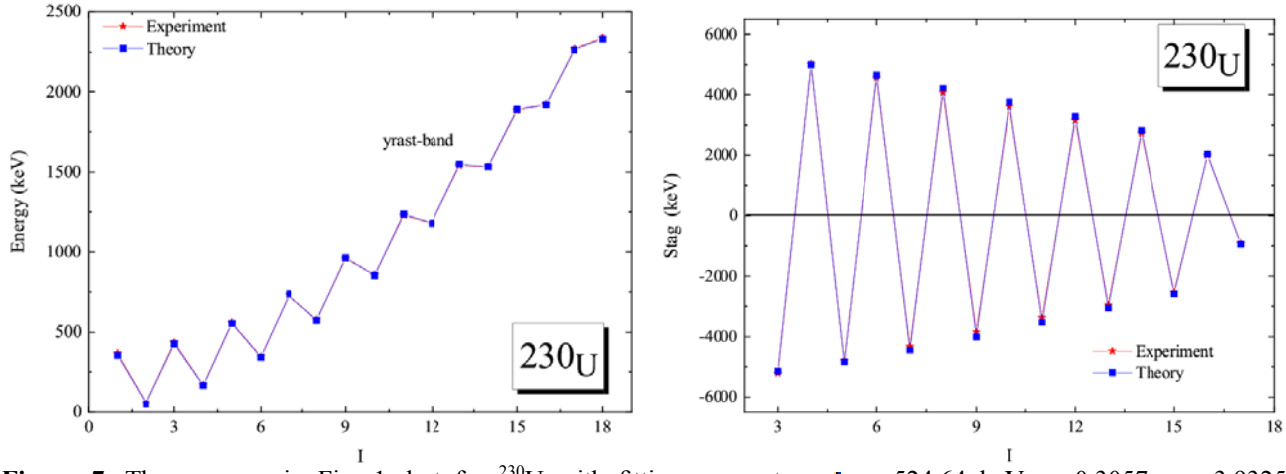


Figure 7. The same as in Fig. 1, but for ^{230}U with fitting parameters: $\hbar\omega_0$=524.64 keV, μ=0.3057, $\varepsilon_{eff}=3.9325^0$, $\gamma_{eff}=52.75^0$, $\eta_{eff}=45.69^0$, RMS=5.87 keV.

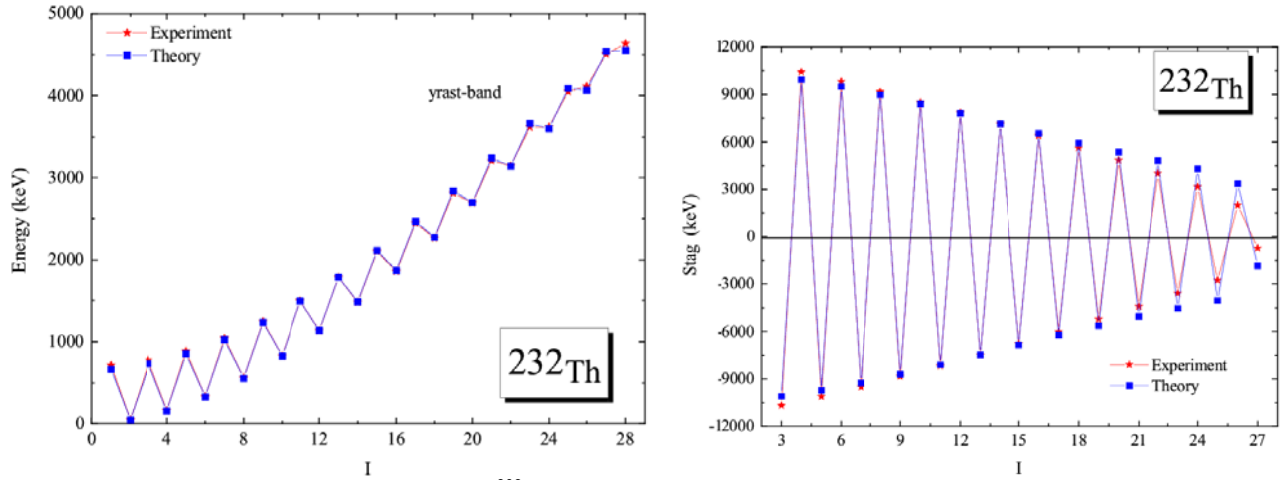


Figure 8. The same as in Fig. 1, but for ^{232}Th with fitting parameters: $\hbar\omega_0$=504.33 keV, μ=0.3078 $\varepsilon_{eff}=0.0001^0$, $\gamma_{eff}=54.7^0$, $\eta_{eff}=14.8^0$, RMS=28.7 keV.

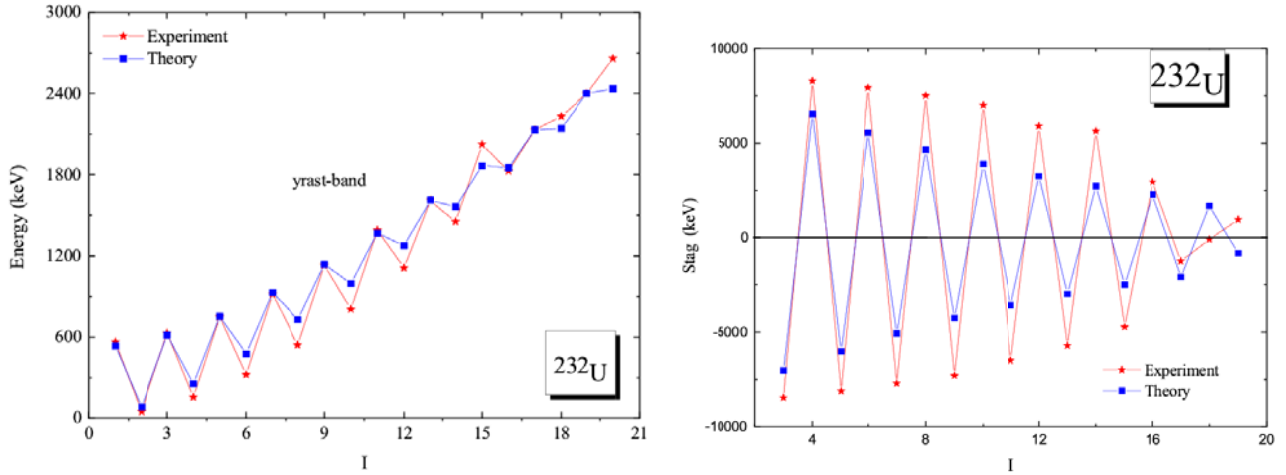


Figure 9. The same as in Fig. 1, but for ^{232}U with fitting parameters: $\hbar\omega_0$=268.23 keV, μ=0.5761, $\varepsilon_{eff}=9.9391^0$, $\gamma_{eff}=56.57^0$, $\eta_{eff}=35.38^0$, RMS=107.17 keV

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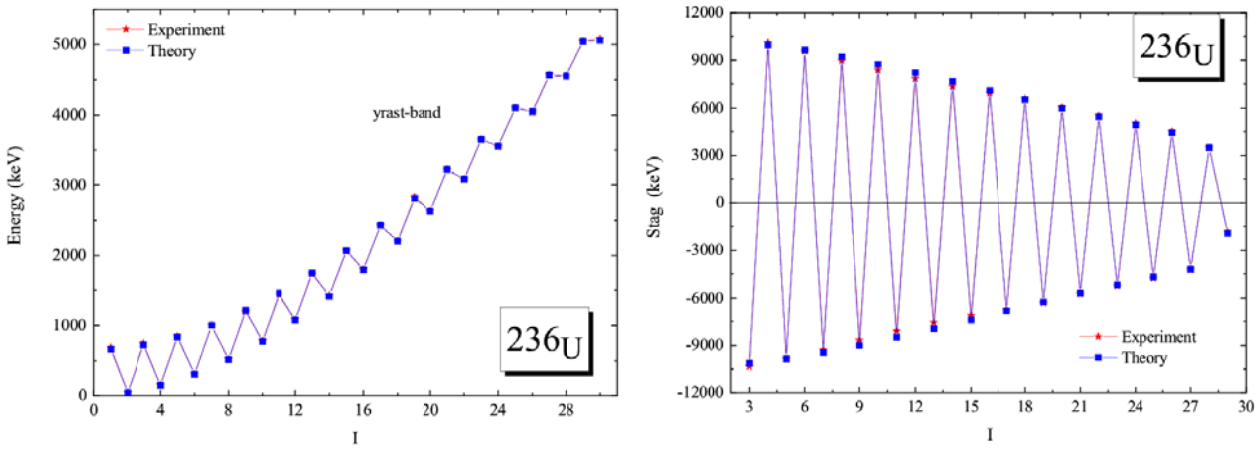


Figure 10. The same as in Fig. 1, but for ^{236}U with fitting parameters: $\hbar\omega_0$=566.06 keV, μ=0.2785, ε_{eff}=0.0062^0, γ_{eff}=54.86^0, η_{eff}=83.88^0, RMS=9.47 keV.

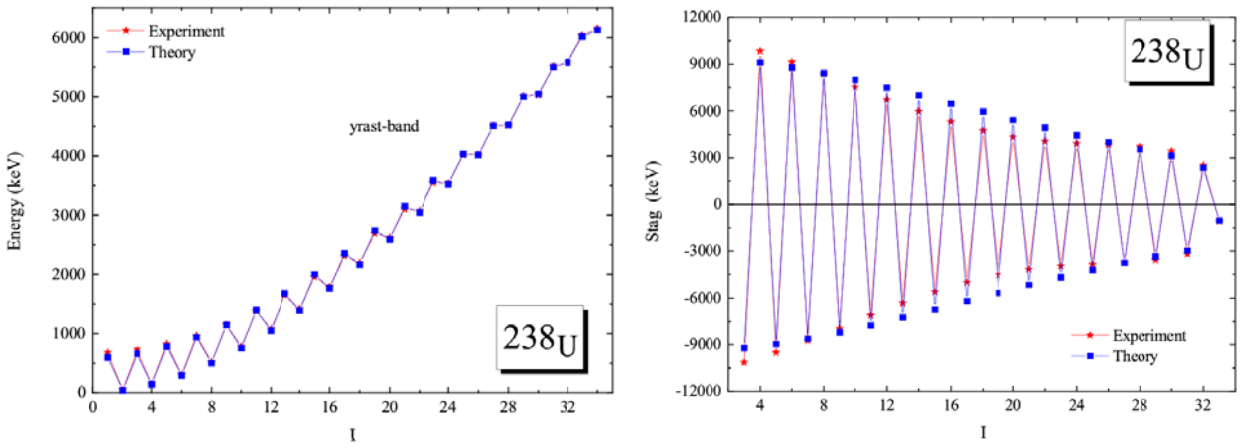


Figure 11. The same as in Fig. 1, but for ^{238}U with fitting parameters: $\hbar\omega_0$=589.89 keV, μ=0.2684, ε_{eff}=0.0003^0, γ_{eff}=54.68^0, η_{eff}=46.79^0, RMS=30.28 keV.

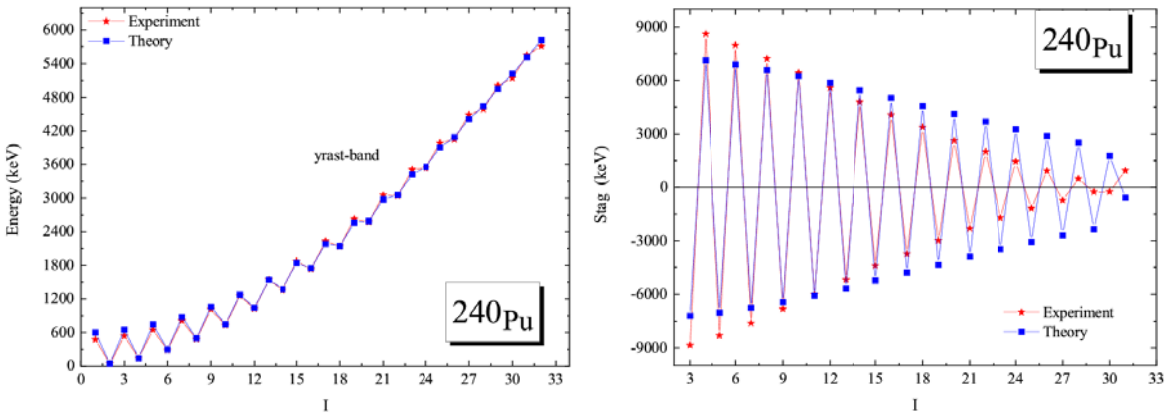


Figure 12. The same as in Fig. 1, but for ^{240}Pu with fitting parameters: $\hbar\omega_0$=698.37 keV, μ=0.24, ε_{eff}=0.0006^0, γ_{eff}=54.11^0, η_{eff}=39.55^0, RMS=59.16 keV

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VI. Odd-even staggering structure

The detailed examination of the quadrupole-octupole collectivity in the considered nuclei on the basis of energy spectra as well as the assessment of model descriptions obtained through both potentials requires a more precise analysis of the APB (alternating parity band) structure [24]

$$\mathrm{Stag}(I) = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2) \qquad (33)$$

with $\Delta E(I) = E(I+1) - E(I)$. The quantity (33) is very sensitive to the fine structure of the rotational spectra and is capable of revealing some rather impressive fluctuations $\Delta I=1$ (zigzag behavior of the discrete high-order energy derivative with respect to reference zero as a function of angular momentum) in various types of energy ranges of rotation and rotational oscillations. In Figs. (1)-(12) (right figs) presents the behavour $\Delta I=1$ staggering-effect.

In the present approach the staggering effect observed in the alternating-parity bands appears as the result of two model features:

i) association of the parity of the observed rotation states (the +/− phases in (21) and (20)) with the parity of the angular momentum values,

ii) mixing the different K-modes due to the assumed triaxial deformations.

We remark that in all considered alternating parity bands, the odd-even staggering does not vanish in the observed region of angular momenta. It is known that the vanishing staggering corresponds to a situation in which the odd and even angular momentum of level sequences (with negative and positive parities) appear ordered in a single nonperturbed collective ”octupole” band. The latter corresponds to the presence of stable quadrupole-octupole deformation. In the presently considered alternating parity bands, however, this does not happen and the observed band-structures are associated with the ”soft” vibrational-rotational quadrupole-octupole deformations.

VII. Conclusion

In this paper, we present a model description of the energy levels of the yrast-band with the alternating-parity spectra of heavy even-even nuclei in the framework of the non-adiabatic collective approach applied in the case of quadrupole-octupole deformations. The Schrödinger equation is solved for the Davidson potential. Analytical expressions for the energy spectrum and wave functions of the collective states of an even-even nucleus are obtained. The model description of the energy levels with alternating parity in the nuclei under consideration shows, on the whole, good agreement with the experimental data.

We note that the nuclei under consideration have a γ-band with large spin. In addition, the simultaneous description of energy levels of alternating and the identical parity, namely descriptions of yrast-, first non-yrast- and γ-bands, are the subject of future work.

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Квадруполь ва октуполь деформацияланган жуфт-жуфт ядроларнинг эффектив ноаксиаллиги

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Квадруполь ва октуполь деформацияланувчи оғир ноаксиал жуфт-жуфт ядроларнинг но-адиабатик коллектив модели ривожлантирилди. Ядро сиртининг квадруполь ва октуполь кўндаланг тебранишлари эффектив, бўйлама тебранишлари эса динамик равишда ҳисобга олинди. Таклиф этилган модель асосида оғир но-аксиал жуфт-жуфт ядроларнинг уйғонган ҳолатлари энергетик сатҳларининг аналатик кўринишдаги ифодаси олинди. Бу ифода назарий ҳисоблашларни экспериментал натижалар билан солиштириш орқали аниқланувчи бешта назарий параметрларга боғлиқ. Қутб координаталарининг бурчак қисми жуфт-жуфт ядронинг мувозанат вазиятини белгилайдиган параметр сифатида фойдаланилди. Ривожлантирилган модель асосида ^{150}Nd, ^{152,154}Sm, ^{172}Yb, ^{228,230,232}Th, ^{232,234,236,238}U ва ^{240}Pu оғир но-аксиал жуфт-жуфт ядроларнинг уйғонган ҳолатларининг yrast-полоса энергетик сатҳлари яхши тавсифланди.

Калит сўзлар: ноадиабатик коллектив модель, квадруполь ва октуполь деформациялар, yrast-полоса.

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