Uzbek Journal of Physics
25(1), 62-66, 2023. DOI: https//doi.org/ 10.52304/.v25i1.407

Coupled dynamics of two impurities in Bose-Einstein condensate

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Received 10.03.2023, revised 26.03.2023.

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В данной работе исследуется динамика двух примесей в квазиодномерном конденсате Бозе-Эйнштейна с учетом влияния амплитуды взаимодействия и начального состояния примеси на связанную динамику примеси и конденсата. Мы рассматриваем взаимодействие между примесями только через конденсат, пренебрегая какими-либо прямыми взаимодействиями. Путем изменения начальных параметров примесей продемонстрирован вклад плотности конденсата в траекторию движения примесей. Результаты показывают, что траектория примесей сильно зависит от их начального положения, что приводит к нерегулярным траекториям из-за деформации плотности конденсата при достаточно сильном притягивающем взаимодействии конденсата с примесями.

Ключевые слова: примеси, конденсат Бозе-Эйнштейна, уравнение Гросса-Питаевского, классическое уравнение Ньютона.

This paper examines the dynamics of two impurities in a quasi-one-dimensional Bose-Einstein condensate, considering the effect of interaction amplitude and the initial state of the impurity on the coupled dynamics of the impurities and condensate. We consider the interaction between impurities only through the condensate neglecting any direct interactions. By changing the initial parameters of the impurities, the contribution of the condensate density to the trajectory of the impurities is demonstrated. The results show that the trajectory of the impurities strongly depends on their initial position, leading to irregular trajectories due to the deformation of the condensate density at a sufficiently strong attractive interaction between the condensate and impurities.

Keywords: impurities, Bose-Einstein condensate, Gross-Pitaevskii equation, classical Newton equation.

I. Introduction

The study of impurities in a Bose-Einstein condensate has been an active area of research in the field of condensed matter physics for several decades [1-2]. Recent experimental advances have enabled researchers to study the motion of heavy impurities or polarons within a Bose-Einstein condensate [3]. The quasiparticles known as polarons find wide-ranging applications in condensed matter physics, including magnetic phenomena and lattice deformation properties [4-6].

The study of impurities in a Bose-Einstein condensate confined by a harmonic potential has been shown to have important implications for understanding the dynamics of Bose-Einstein condensates in a variety of experimental setups [7, 8].

The dynamics of the system are described by the Gross-Pitaevskii equation for the condensate and the classical Newton equation for the impurities [9]. The Gross-Pitaevskii equation is a mean-field theory that describes the dynamics of a Bose-Einstein condensate in terms of a macroscopic wave function. The classical

Newton equation describes the motion of the impurities in the system.

The motion of heavy impurities in a Bose-Einstein condensate has been studied extensively in recent years. A number of studies have focused on the formation and dynamics of polarons in a Bose-Einstein condensate [10-17]. Other studies have focused on the dynamics polaron forming impurities in a quasi-one- and two-dimensional Bose-Einstein condensate confined by a harmonic potential [18-21]. In addition, the motion of impurities in a Bose-Einstein condensate has been studied in the context of quantum simulations and quantum information processing [22].

In this paper, we study the coupled motion of two heavy impurities attractively interacting only through a quasi-one-dimensional Bose-Einstein condensate confined by a harmonic potential. We use a direct numerical solution of the system of Gross-Pitaevskii and classical Newton equations to study the dynamics of the system. We show that the motion of the attractive interacting two impurities and the condensate is strongly coupled and depends critically on the initial parameters of the system, namely on the initial positions of the impurities and interaction amplitude. Our results demonstrate the importance of understanding the dynamics of heavy directly non coupled impurities in a Bose-Einstein condensate and the effect of the initial parameters on the motion of the system.

II. Model and equations of motion

We consider non-self-interacting quasi-one-dimensional Bose-Einstein condensate characterized by wave function $\psi(x,t)$ located in a parabolic potential. We assume that two impurities formed by externally embedded particles have interaction only through the condensate. In what follows we set, $\hbar \equiv m \equiv \omega \equiv 1$, where $\hbar$ is a reduced Plank constant, m is condensate particle mass and ω is the frequency of the harmonic potential. It is assumed that the $\psi(x,t)$ is normalized to the total number of particles in the condensate $N>>1$. The system is described by the Gross-Pitaevskii equation,

$$i\frac{\partial}{\partial t}\psi(x,t)=\left(-\frac{1}{2}\frac{\partial^2}{\partial x^2}+\frac{x^2}{2}\right)\psi(x,t)+\mathcal{V}_{\text{ext}}(x)\psi(x,t)\,, \tag{1}$$

where

$$\mathcal{V}_{\text{ext}}(x)=\mathcal{V}_1\left(x-X_1(t)\right)+\mathcal{V}_2\left(x-X_2(t)\right).$$

The $\mathcal{V}_1(x-X_1(t))\psi(x,t)$ and $\mathcal{V}_2\left(x-X_2(t)\right)\psi(x,t)$ terms describe the local interaction of the BEC with embedded movable impurity located at the time-dependent positions $X_1(t)$ and $X_2(t)$. We assume that the impurities have the same masses M and potentials. In what follows we choose a narrow attracting potential $\mathcal{V}_i(z)=V\exp\left(-z_i^2/2\delta^2\right)$, where $V<0$ is the amplitude and δ is the width, common for both embedded particles.

We consider sufficiently heavy, with respect to the condensed particles, embedded particles such that the evolution of their positions is described by the classical Newton equation:

$$M\frac{d^2X_i(t)}{dt^2}=\mathcal{F}_c^{[i]}(X_i(t),X_j(t)). \tag{2}$$

The time-dependent force between the condensate and the impurity i depends on the positions of both the particles and has the form [16]:

$$\mathcal{F}_c^{[i]}(X_i(t),X_j(t))=-\partial_{X_i(t)}\int_{-\infty}^{\infty}\mathcal{V}_{\text{ext}}(x)\,|\,\psi(x,t)|^2\,dx, \tag{3}$$

Here, the interaction with condensate produces a relatively small albeit well-visible effect. Please note that when impurities are present, their interaction with each other can be approximated as an interaction via the condensate. This is because the presence of an impurity particle at point $X_j(t)$ modifies the condensate density at the position of another impurity particle at $X_i(t)$ point. The system of equations (1)-(3) describes all dynamics parameters of the system.

III. Expansion by basis function and the energy of the system

To obtain a numerical solution of the Gross-Pitaevskii equation, we express the wave function $\psi(x,t)$ of the condensate as a series expansion of the eigenfunctions in relation to the parabolic potential. This potential eigenfunctions are indexed by the variable k and is used as the basis for the expansion:

$$\psi_k(x)=\frac{H_k(x)}{\pi^{1/4}(2^k k!)^{1/2}}\exp\left(-\frac{x^2}{2}\right), \tag{4}$$

where, $H_k(x)$ being the corresponding Hermite polynomial. The wave function describing BEC is presented in the form of the expansion in the sufficient truncated basis as:

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$$\psi(x,t)=\sum_{k=0}^{n}A_k(t)\psi_k(x), \tag{5}$$

by $n+1$ eigenfunctions with time-dependent coefficients normalized by $\sum_{k=0}^{n}\left|A_k^2(t)\right|=N$. Below for simplicity we omit some explicit $t-$ dependencies when it will not cause confusion.

Applying wavefunction (5) expanded by basis functions (4) in partial differential equation (1) and integrating by overall coordinate space we obtain ordinary differential equation depending on the evolution of the expansion coefficients as:

$$i\frac{\partial A_k}{\partial t}=A_k\omega\left(k+\frac{1}{2}\right)+\sum_{k_1}A_{k_1}\int_{-\infty}^{\infty}\psi_k(x)\mathcal{V}_{\text{ext}}(x)\psi_{k_1}(x)dx, \tag{6}$$

where k is the corresponding set of the states.

The form of the energy of the condensate when interacting with the embedded particles is:

$$E_c=\sum_{k=0}^{n}|A_k|^2\left(k+\frac{1}{2}\right)+ +\sum_{\{k\}}A_{k_1}^{*}A_{k_2}\int_{-\infty}^{\infty}\psi_{k_1}(x)\mathcal{V}_{\text{ext}}(x)\psi_{k_2}(x)dx. \tag{7}$$

The set of equilibrium states $A_k(0)$ is established through a process of energy minimization, which differs from the microscopic variational procedure described in Ref. [23] and more corresponds to the problem of the ground state of the condensate [24-25]. This procedure involves minimizing the energy of the BEC when interacting with the embedded particles located at the initial positions $X_1(t)$ and $X_2(t)$.

Now applying the condensate wave function (5) in the classical Newton equation (2) we obtain the contribution of the condensate by a set of coefficients $A_k(0)$ into the coordinate of the embedded particle in the form:

$$M\frac{dX_i(t)}{dt}=-\partial_{X_i(t)}\sum_{\{k\}}A_{k_2}A_{k_1}^{*}\int_{-\infty}^{\infty}\psi_{k_1}(x)\mathcal{V}_{\text{ext}}(x)\psi_{k_2}(x)dx. \tag{8}$$

We obtain a closed system of first-order nonlinear differential equations and solve it numerically. To ensure the accuracy of the numerical results, we use a truncated basis of $n+1$ states, where n is the highest number of states required to maintain the consistency of the results. In our case, we find that a truncated basis of 21 states is sufficient for the systems under consideration.

IV. Numerical results and discussions

With the numerical simulation we will demonstrate trajectories of impurities $X_i(t)$ and a fidelity of condensate:

$$F(t)=\left|\int_{-\infty}^{\infty}\psi^{*}(x,t)\psi(x,0)dx\right|^2=\left|\sum_{k=0}^{n}A_k^{*}A_k(0)\right|^2, \tag{9}$$

characterizing the change in shape and position of the condensate by following evolution of the $A_k(t)$ components. Also, we will present density deformation of the condensate from the initial state defined by

$$D(x,t)=|\psi(x,t)|^2-|\psi(x,0)|^2. \tag{10}$$

In a numerical calculation we consider a mass of the impurities only the case when $M = Nm$, that means mass of the impurity equal to total mass of condensate and narrow width potential for $\delta = 0.1$. Our primary objective is to compare the influence of the interaction amplitude V and initial state $X_i(0)$ of the impurity on the dynamics of both the impurities and condensate.

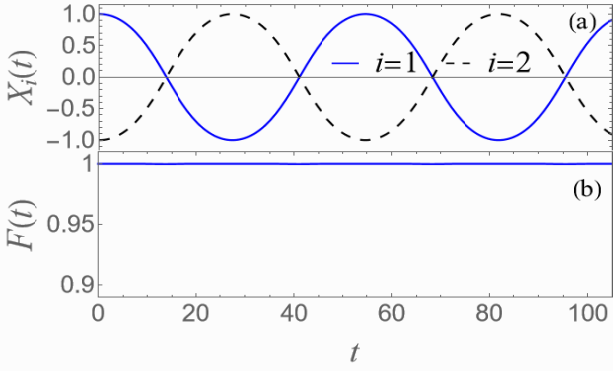


Figure 1. Plot of trajectories of the impurities (a) and fidelity of the condensate (b) defined by Eq. (9) for given initial parameters of impurity as: V=–0.1, and $X_1(0)$=1, $X_2(0)$=–1.

Figure 1(a) presents the trajectories of two impurities, (b) is fidelity of condensate for given initial parameters. From Fig. 1(b) one can see that for V=–0.1 interaction between condensate and impurities are considerably week to change condensate density and to affect to trajectories of its. Therefore, fidelity is constant and in the Fig. 1(a) two impurities are oscillating independently to the positions of each other. Now, we increase the amplitude of potentials of impurities to V=–0.1, that can lead deformation of

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ground states of parabolic potentials.

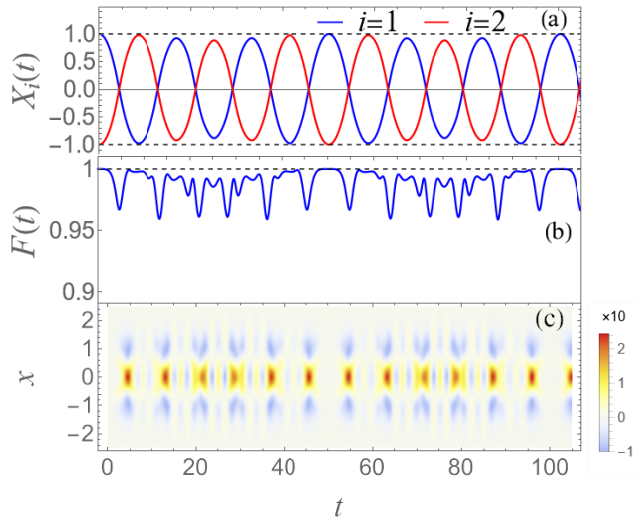


Figure 2. Plot of (a) trajectories of the impurities, (b) fidelity of the condensate defined by Eq. (9) and density deformation of the condensate defined by Eq. (10) for given initial parameters of impurity as: V=–0.1, and $X_1(0)$=1, $X_2(0)$=–1.

Comparing Figs 1(a) and 2(a) one can see that by increasing in V, oscillation frequency of impurities in a condensate are increased. On the other hand, Fig. 2(b) it clearly shown that the fidelity is decreased with the course of the oscillation frequency of impurities in Fig. 2(a). In addition, the dynamics of deformation Fig. 2(c) of the condensate demonstrates how the fidelity of condensate changed. When two impurities are in the center of condensate deformation of condensate density Figs 2(a), (b) and (c) behave as consistently, because when two impurities $X_i(t)$ are located in a minimum of the parabolic potential, then the density of condensate is strongly attracted by impurities and increased in the center and decreased at the edge of condensate consequently. Also, the fidelity is decreased to the minimum value which is about 5%. As a result, we have found that the dynamics of two symmetrically embedded particles in the condensate strongly depend on the attractive interaction between particles and condensate.

Now we assume that the second impurity $X_2(0)$ initially embedded in the center of the condensate and $X_1(0)$ is at some distance from the center. Then the displacement of impurity $X_2(t)$ strongly coupled with the motion of impurity $X_1(t)$ due to the interaction through the condensate.

From Fig. 3(a), it follows that at a short time the impurity $X_1(t)$ starts to move toward the origin of coordinates and, at the same time, due to the large mass, the impurity attracts the condensate to itself.

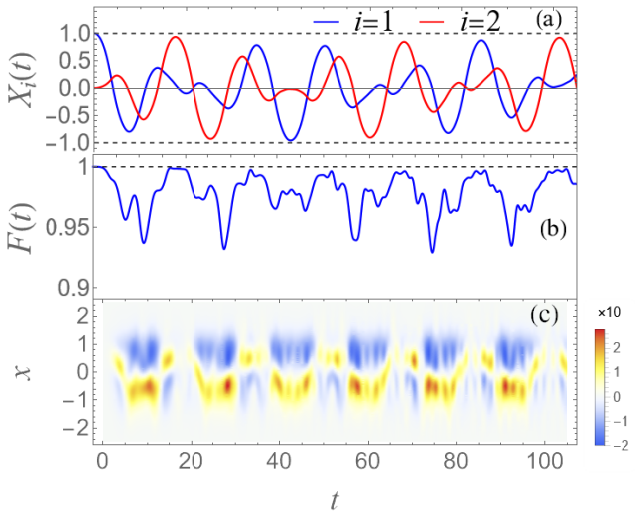


Figure 3. Plot of (a) trajectories of the impurities, (b) fidelity of the condensate defined by Eq. (9) and density deformation of the condensate defined by Eq. (10) for given initial parameters of impurity as: V=–1, $X_1(0)$=1 and $X_2(0)$=0.

In this time shifted condensate attracts the impurity $X_2(t)$ located in the center of the condensate toward the first $X_1(t)$ impurity, thus appears nontrivial coupled dynamics of the two impurities interacting through the condensate. Fig. 3(a), (b) and (c) show that the trajectories of two impurities strongly depend on position of each other and deformation of the condensate which is strongly become irregular. Since, the initial density distribution of the condensate is not symmetric and shifted to the positive side from the origin of the coordinate, therefore Fig. (c) shows that the deformation of density mostly positive in negative coordinate and negative in positive coordinate. At some fixed point in time, the impurities return to their original position, and the deformation of the condensate vanishes. As a result, we found that the dynamics of two asymmetrically embedded impurities in the condensate and interaction through the condensate strongly depend on the position of each other and the deformation of the condensate.

V. Conclusion

In this study, we investigate the coupled dynamics of two identical heavy impurities embedded in a Bose-Einstein condensate that is confined by a harmonical potential. We consider only the attractive interaction between the condensate and the impurities

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while neglecting the self-interaction of the condensate. By varying the initial parameters of the impurities, such as the amplitude of interaction with the condensate and the initial position, we aim to demonstrate the contribution of the condensate density to the trajectory of the impurities.

Our results reveal that at sufficiently strong attractive interaction between the condensate and the impurities that can alter the initial state of the condensate, the trajectory of the impurities becomes highly dependent on their initial position, leading to irregular trajectories due to the deformation of the condensate density.

Acknowledgments. This research is supported by Grant F-FA-2021-432 of the Uzbekistan Agency for Innovative Development. The author are grateful to professors B.J. Ahmedov and E.Ya. Sherman for the valuable discussions.

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Босе-Эйнштейн конденсатида иккита ўзаро таъсирдаги аралашмаларнинг динамикаси

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Ушбу мақолада квази-бир-ўлчовли Бозе-Эйнштейн конденсатидаги иккита аралашмаларнинг динамикасини ўрганилди. Бунда аралашмалар ўртасидаги таъсир фақат конденсат орқали мавжуд ва тўғридан-тўғри таъсир мавжуд эмас деб қаралди. Конденсат ва аралашма ўртасидаги ўзаро таъсир кучсиз ва кучи тортишунчан бўган ҳолатлар ўрганилди. Асосан, аралашмаларнинг дастлабки параметрларини ўзгартириш орқали конденсат зичлигининг аралашмалар траекториясига таъсири кўрсатилади.

Натижалар шуни кўрсатадики, аралашмаларнинг траекториялари уларнинг дастлабки параметрларига кучли боғлиқ экан. Айниқса конденсат ва аралашмалар ўртасидаги тортишувчанлик конденсат зичлигини ўзгартирадиган даражада кучли бўлганда ва аралашмалар бошланғич жойлавув туқталари ҳар-хил бўганда деформацияланган конденсат зичлиги аралашламаларнинг тартибсиз траекторияларга олиб келиши кўрсатилди.

Калит сўзлар: аралашма, Бозе-Эйнштейна конденсати, Гросс-Питаевский тенглама, Ньютон классик тенглама.

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